OpRisk NorthAmerica

17th Annual Conference
The biggest annual gathering of
operational risk professionals
March 23-26 2015, New York

Extreme Losses and Operational Risk Capital: Myths and Realities

J.D. Opdyke
Head of Operational Risk Modeling
GE Capital

Disclaimer

The views presented in this presentation are the views of the sole author, J.D. Opdyke, and not necessarily the views of GE Capital. Any results presented herein are based on publicly available sources of information.

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I. Extreme OpRisk Losses

Operational Risk
- Basel II/III
  - Advanced Measurement Approach
    - Risk Measurement & Capital Quantification
      - Loss Distribution Approach
        - Frequency Distribution
        - Severity Distribution* (by far the main driver of the aggregate loss distribution)

* For purposes of this presentation, potential dependence between the frequency and severity distributions is ignored. See Ergashev (2008) and Chernobai, Rachev, and Fabozzi (2007).

I. Extreme OpRisk Losses

- <u>**The (Compound) Aggregate Loss Distribution**</u>

  Under the Loss Distribution Approach, by far the most widely implemented of the Advanced Measurement Approaches (see BCBS, 2004), operational risk capital is based on the Aggregate Loss Distribution (ALD).

- The ALD is the convolution of two estimated loss distributions: the frequency distribution, representing the number of operational risk loss events that can take place during a given timeframe (typically a year), and the severity distribution, representing the size of these loss events.

- Regulatory Capital (RCap) is the estimated 99.9 Value-at-Risk (VaR) of the ALD: in other words, the quantile ($ amount) associated with the 99.9%tile of the ALD.

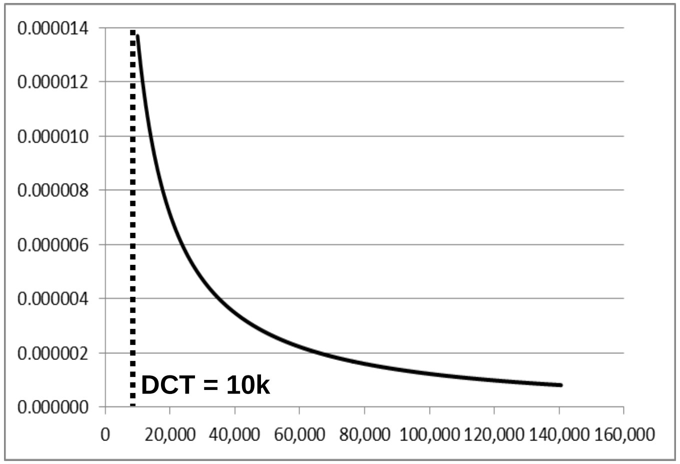
- Operational Risk Loss Event data is separated into exhaustive and (generally) mutually exclusive cells, or Units of Measure (UoM's), defined so that the loss distributions associated with each are as homogenous as possible (i.e. all coming from one frequency and one severity distribution, or close). Often the Business Lines, Event Types, or some combination of both define a financial institution's UoM's.

- The 99.9%tile of each UoM is estimated. Because losses at all the 99.9%tiles do not occur in perfect lockstep, they should not simply be added to obtain RCap at the enterprise level: dependence structure across the UoM's is estimated, and the decrease in Enterprise Level Capital (relative to a simple sum) that results from this diversification benefit typically is substantial (decreases of 25-50% are not uncommon; see RMA, 2011; OR&R, 2009; and Haubenstock & Hardin, 2003).

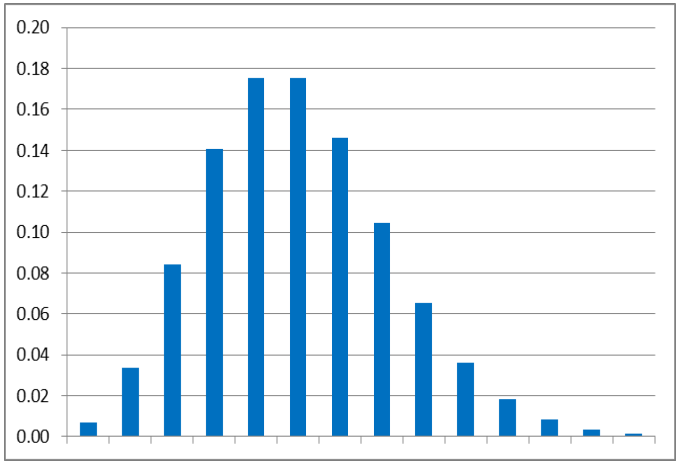
I. Extreme OpRisk Losses

For a given UoM:

Estimated Severity PDF – Truncated LogNormal (μ=10, σ=2.8, H=10k)

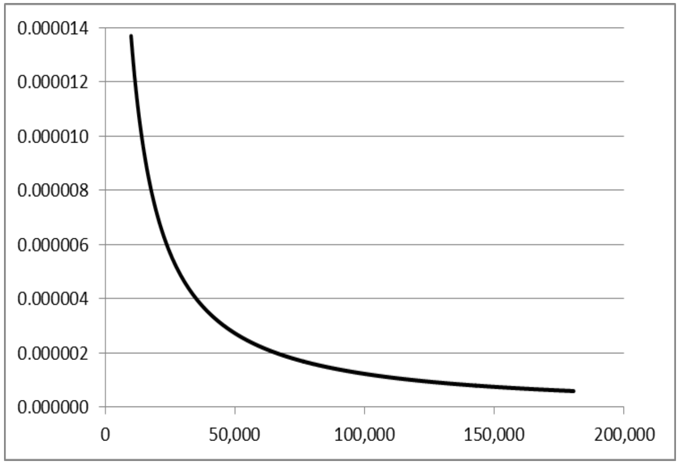


Estimated Frequency PMF – Poisson (annual λ=25)



Convolution to combine frequency and severity into ALD (in practice, rarely a closed form solution … but for the VaR there are good and widely accepted analytical approximations to avoid extensive simulation)

Aggregate Loss Distribution (ALD)



Regulatory Capital = VaR at 99.9%tile

II. Only 2 Fundamental Problems

i. **Excessive Capital Variability**,and
ii. **Inflated/Large Capital Size**

are the Two Fundamental Challenges facing AMA Operational Risk Capital Estimation under a Loss Distribution Approach.

III. Variability of Capital

Excessive Variability in the Operational Risk Capital Estimate

i. This subsumes almost all issues related to “Model Robustness”, “Model (In)Stability” etc. (because if this cannot be mitigated, no other mitigation efforts related to “robustness” or “stability” will work).

ii. Simply put, AMA Operational Risk Capital Estimates generated under a Loss Distribution Approach simply are not precise enough to be useful. When capital of $500M is associated with (95%) confidence intervals ranging (<u>optimistically</u>) from $50M to $1,500M, we can put little confidence in decisions based on them.

iii. This EXCESSIVE VARIABILITY is a function of five things:

III. Variability of Capital

Excessive Variability in the Operational Risk Capital Estimate

iii. This EXCESSIVE VARIABILITY is a function of six things:

"old"

a. data paucity and heterogeneity

b. the EXCESSIVELY high severity percentile that must be estimated based on the 99.9%tile of the Aggregate Loss Distribution

c. All relevant severities are heavy-tailed

"new"

d. High Probability, Low-Severity Loss Events (Yes, you read this right)

e. "behavioral convexity" of VaR as a function of severity parameter estimates

f. Systemically Upward Capital Bias (inflation) due to an essential discontinuity in the most commonly used quantile approximation: Degen's (2010) Single Loss Approximation (SLA)

III. Variability of Capital

Table 1:
RCE vs. LDA-MLE for Truncated LogNormal Severity (μ = 10.7, σ = 2.385, H=$10k)*
[see Opdyke (2014) for complete results]

(millions)	Regulatory Capital**			Economic Capital**	
	RCE	LDA-MLE		RCE	LDA-MLE
Mean*	$700	$847		$1,338	$1,678
True Capital	$670	$670		$1,267	$1,267
Bias (Mean - True)	$30	$177		$71	$411
Bias %	4.5%	26.4%		5.6%	32.4%
RMSE*	$469	$665		$1,003	$1,521
STDDev*	$468	$641		$1,000	$1,464

* 1,000 Simulations, n ≈ 250

** λ = 25; α = 0.999 RC; α = 0.9997 EC

III. Variability of Capital

Table 2:
RCE vs. LDA-MLE for GPD Severity (ξ = 0.875, θ = 47,500, H=$0k)*
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(millions)	Regulatory Capital**			Economic Capital**	
	RCE	LDA-MLE		RCE	LDA-MLE
Mean*	$396	$640		$1,016	$2,123
True Capital	$391	$391		$1,106	$1,106
Bias (Mean - True)	$5	$249		$24	$1,016
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III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

a. <u>data paucity and heterogeneity</u>:

unfortunately, increasing sample size will only marginally improve precision in the capital estimate (see Shevchenko, 2011), and OpRisk Loss event data always will be relatively heterogeneous no matter how well we define UoMs. OpRisk loss events are by their nature very diverse. The smaller we make UoMs, the more homogenous they are, but then we lose statistical power (precision) in our capital estimates. This is an absolutely unavoidable tradeoff between homogeneity and statistical power.

III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

a. **<u>data paucity and heterogeneity</u>**:

<u>MYTH</u>: If we have a large enough sample size of loss events, we can get reasonably precise capital estimates.

<u>REALITY</u>: Even under idealized, textbook loss event data (i.i.d. data), we would have to have 50,000 to 100,000 years of loss events to even approach reasonably precise capital estimates. Larger sample sizes, per se, will only very marginally improve precision in the capital estimate.

III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

b. **<u>the EXCESSIVELY high severity percentile</u>** that must be estimated based on the 99.9%tile of the Aggregate Loss Distribution

Take Degen's (2010) widely used Single Loss Approximation as a heuristic.

$$C_\alpha \approx F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)+\lambda\mu \quad \text{and} \quad F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right) >> \lambda\mu$$

so this is essentially an estimate of an extremely high quantile of the severity distribution, because the 99.9%tile VaR of the Aggregate Loss Distribution ($\alpha = 0.999$) corresponds to a MUCH HIGHER %tile of the severity distribution. For example, if $\lambda = 30$, then

$$\left(1-\frac{1-\alpha}{\lambda}\right) = 0.999967$$

so a 99.9%tile of ALD is a 99.9967%tile of the severity!

III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

b. **the EXCESSIVELY high severity percentile** that must be estimated based on the 99.9%tile of the Aggregate Loss Distribution

so a 99.9%tile of ALD is a 99.9967%tile of the severity!

The higher the %tile, the greater the variability in the estimation of that %tile.

Intuitively, imagine taking one end of a long 2x6 and shaking it hard: you'll see very little "wobble" (variation) in the middle of the board.

Now imagine watching the END of the 2x6 as we're shaking it: the "wobble" will be considerably larger.

III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

b. **the EXCESSIVELY high severity percentile** that must be estimated based on the 99.9%tile of the Aggregate Loss Distribution

so a 99.9%tile of ALD is a 99.9967%tile of the severity!

This is analogous to estimating two %tiles of the Aggregate Loss Distribution based on samples. Every time we "shake" the board we are drawing another sample from the data generating mechanism. If we try to mark where the midpoint of the long 2x6 is, i.e. the 50%tile, we can do this pretty accurately because the 2x6 is not bouncing around as much in the middle. But if we try to mark where the 99.9%tile is at the END of the long board, this is much harder because the end is bouncing around so much more.

AND note that this variation in the estimate of the %tile will increase nonlinearly as the %tile increases.

III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

  b. **the EXCESSIVELY high severity percentile** that must be estimated based on the 99.9%tile of the Aggregate Loss Distribution

MYTH: We are (only) estimating the 99.9%tile of the Aggregate Loss Distribution.

REALITY: We are actually estimating the 99.9967+%tile of the severity distribution. And percentiles of 99.999%tile and higher are not uncommon. This is often two orders of magnitude larger than the 99.9%tile, making this what many statisticians would call an ill-posed problem, ESPECIALLY under data conditions that are far from ideal (i.e. far from independent and identically distributed, or i.i.d.).

III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

c. **All relevant severities are Heavy-Tailed**

Severity distributions that are not at least fairly heavy-tailed are not going to pass regulatory muster.

When estimating the same high %tiles, those of heavier-tailed severity distributions often are associated with greater variability vs. those associated with lighter-tailed distributions.

However, even though truncation unambiguously makes severity tails heavier, its affect on capital variability is to mitigate it somewhat (as shown in the next section).

III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

d. **High Probability, Low-Severity Loss Events**
(Yes, you read this right)

For almost all severity estimators used in this space, maximum likelihood being by far the most common (see RMA, 2013), excessive quarter-to-quarter variability in the capital estimate is NOT driven by Low Probability, High Severity losses. It is driven by High Probability, Low Severity losses.

This is demonstrated by the mathematics of the Influence Function (see Opdyke and Cavallo, 2012a, 2012b).

Truncation of the severity mitigates, but does not eliminate this problem. The **EXCESSIVE SENSITIVITY of the capital estimate to HIGH PROBABILITY, LOW SEVERITY LOSSES** is still very material under truncated severities.

III. Variability of Capital

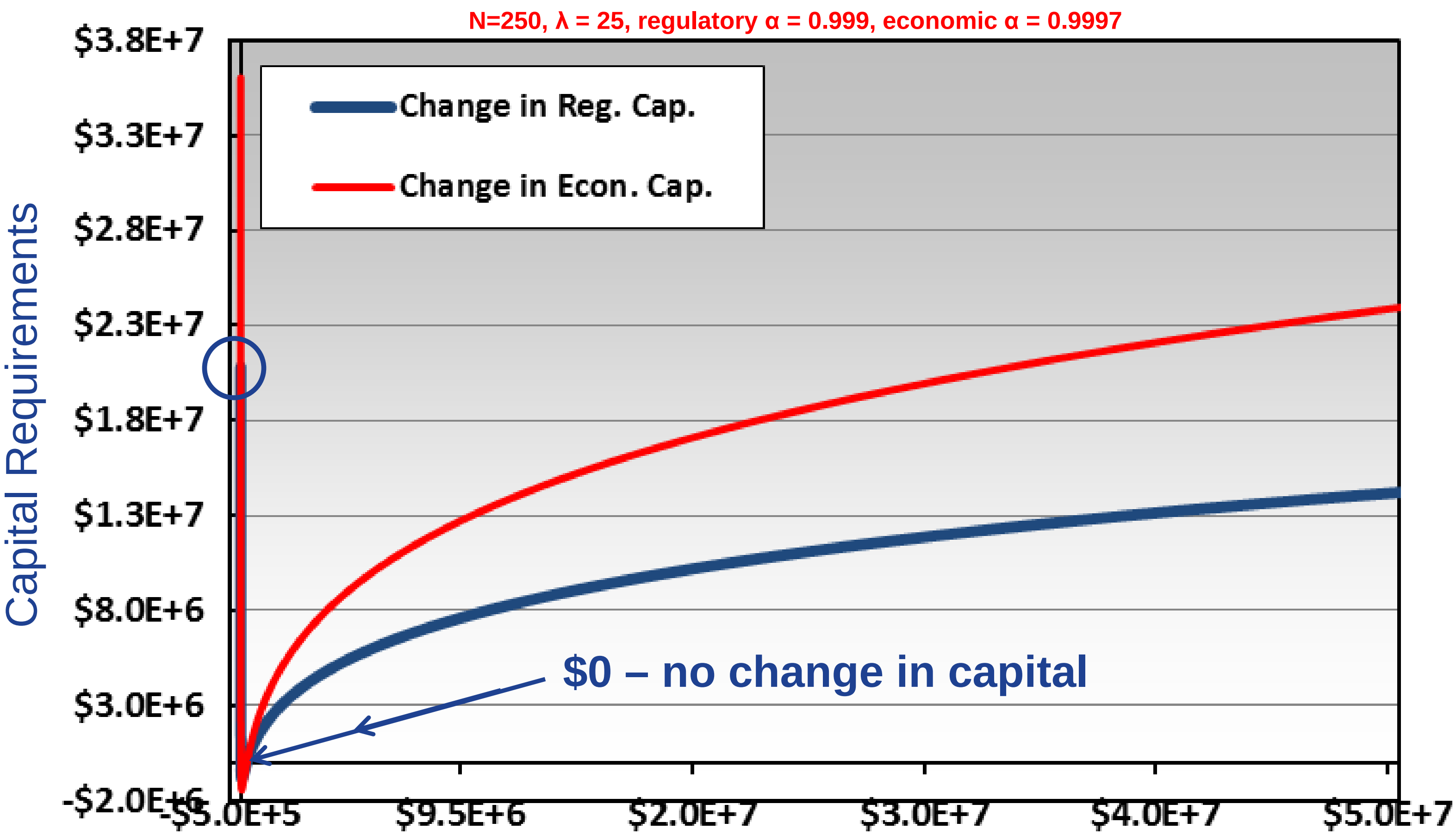
- These extreme capital responses to small, left-tail losses are not just mathematical curiosities: they are possibly the largest source of quarter-to-quarter instability of MLE-based capital requirements, because they are not as rare as "low frequency, high severity" losses. The effects are still extreme even for losses within $4k of the lower threshold, losses that every bank has in its severity modeling loss event datasets.

Severity	Threshold	Parameter			Change in Capital ($mill)					
					H + $10 loss		H + $2k loss		H + $4k loss	
Dist.	H	Names	Parm 1	Parm 2	RC	EC	RC	EC	RC	EC
LogN	$0	μ, σ	10.953	1.749	$19.0	$33.3	$1.3	$2.4	$0.4	$0.8
LogN	$10,000	μ, σ	10.954	1.750	$2.6	$4.2	$2.0	$3.6	$1.5	$2.4
LogN	$25,000	μ, σ	10.917	1.749	$2.6	$4.8	$2.3	$4.2	$2.0	$3.6
LogG	$0	α, β	35.484	3.252	$590.9	$1,469.8	$14.1	$34.1	$3.6	$9.2
LogG	$10,000	α, β	35.513	3.263	$24.1	$62.2	$18.0	$43.1	$13.2	$33.5
LogG	$25,000	α, β	35.410	3.252	$26.4	$67.0	$22.8	$57.4	$19.2	$57.4
GPD	$0	ξ, β	0.8713	57,584	$27.9	$92.2	$24.0	$79.5	$20.4	$67.8
GPD	$10,000	ξ, β	0.8825	57,484	$31.2	$95.6	$26.4	$95.5	$24.0	$76.4
GPD	$25,000	ξ, β	0.8798	57,340	$38.4	$133.8	$36.0	$133.7	$31.2	$95.5

- All it takes is a couple of new losses near the threshold, or changes in the values of such existing losses, to induce dramatic variability and instability in MLE-based capital requirements from quarter to quarter.

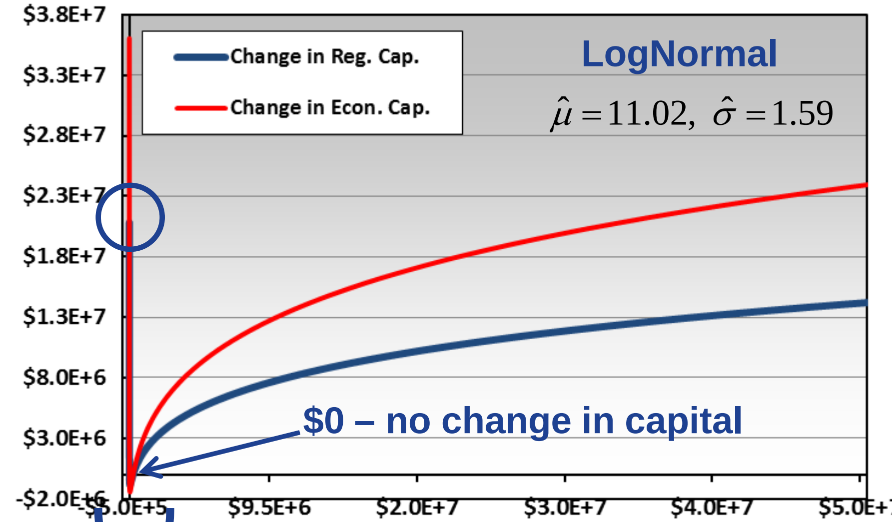
III. Variability of Capital

Based on a Random Draw from LogNormal (μ=10.95, σ=1.75) where MLE $\hat{\mu} = 11.02,\ \hat{\sigma} = 1.59$

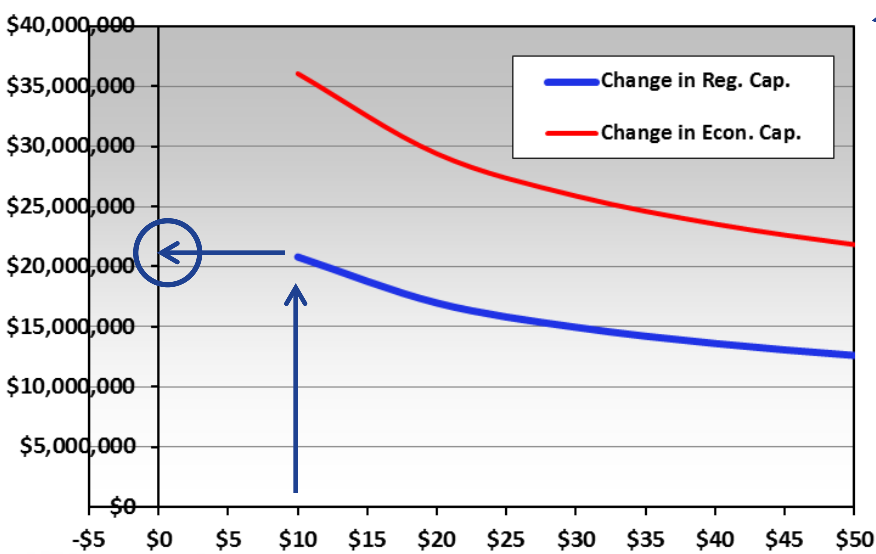


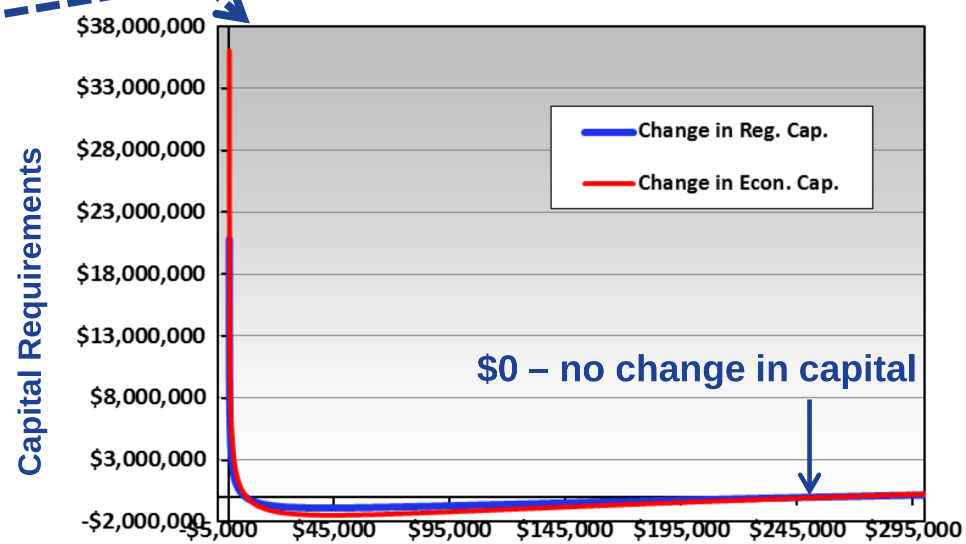
III. Variability of Capital

For MLE, a new loss of $10 increases regulatory capital by over $20m, and economic capital by over $36m. But a loss of about $250k increases capital by $0.



N=250, λ = 25, regulatory α = 0.999, economic α = 0.9997

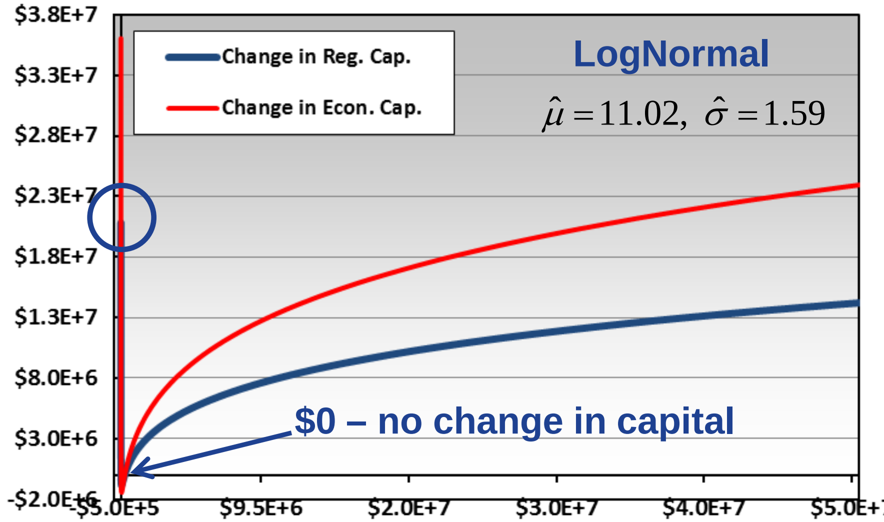




III. Variability of Capital

WHY? Check the MLE IF, which we derived previously as:

$$IF_{\theta}\left(x;\theta,T\right)=\begin{bmatrix}\ln\left(x\right)-\mu\\ \dfrac{\left(\ln\left(x\right)-\mu\right)^{2}-\sigma^{2}}{2\sigma}\end{bmatrix}$$

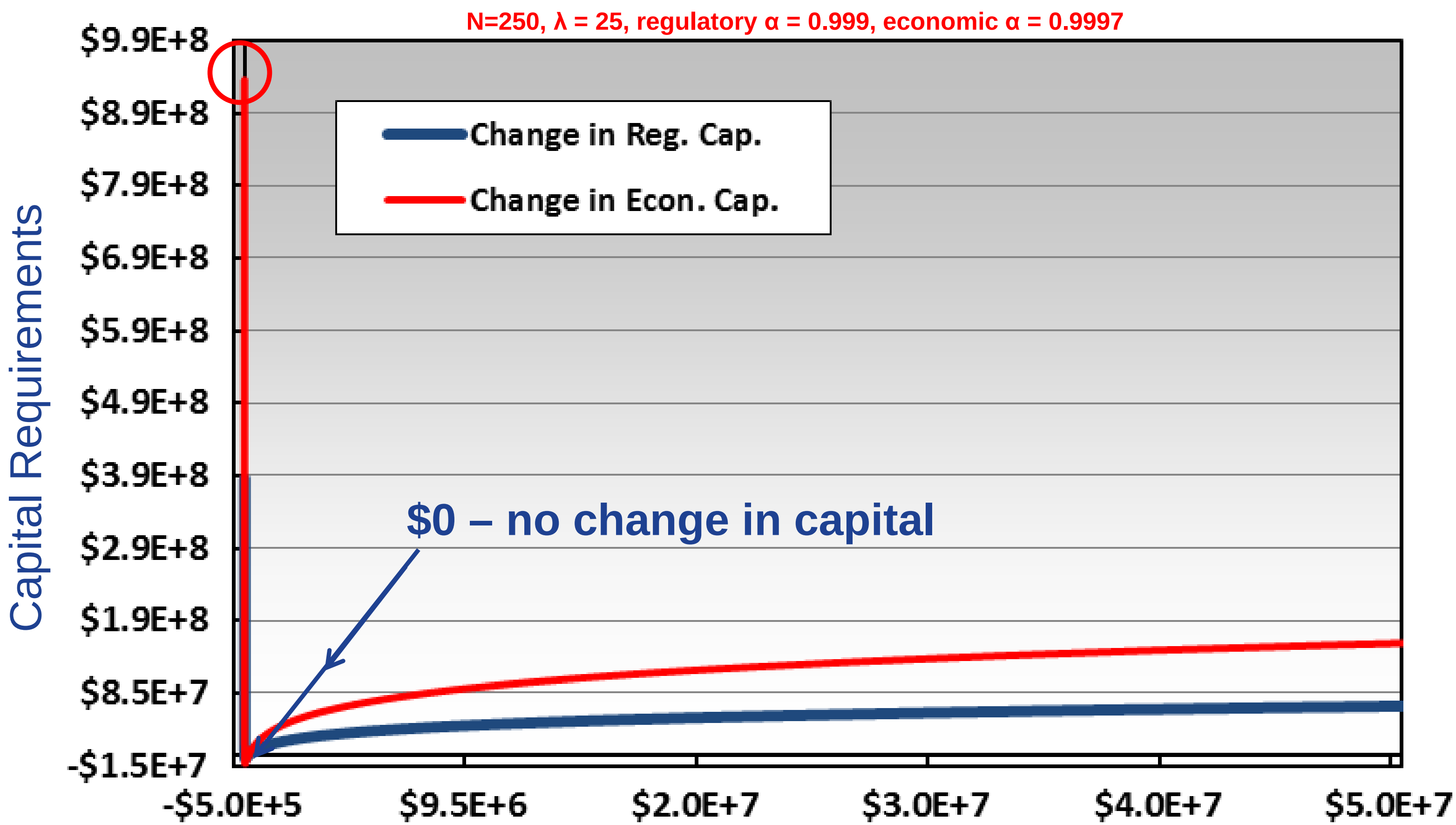


N=250, λ = 25, regulatory α = 0.999, economic α = 0.9997

The IF for the σ term becomes HUGE when $x \rightarrow 0^{+}$, so required capital also is going to become HUGE as it is based directly on the HUGE parameter estimate for σ. Even though the IF indicates that the parameter estimate for μ decreases monotonically as x decreases, it does so at a much slower rate so the effect of σ will dominate the effect that x has on capital.

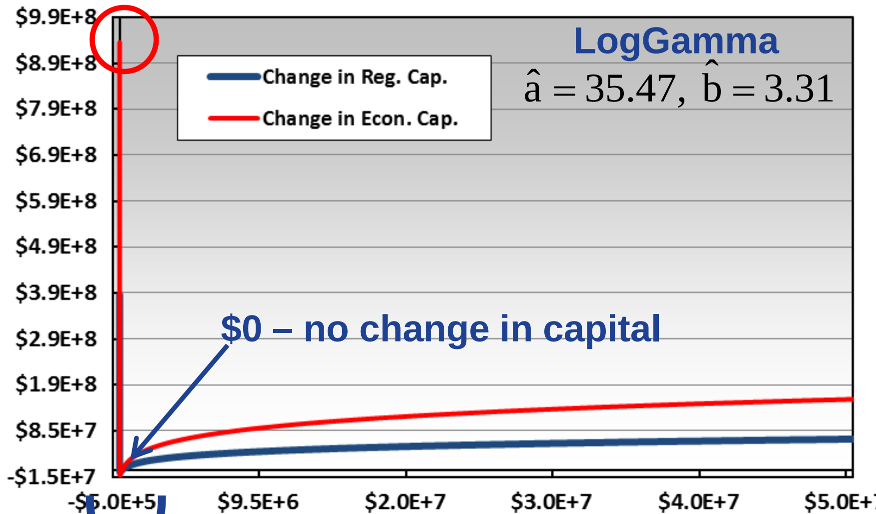
III. Variability of Capital

Based on a Random Draw from LogGamma (a=35.5, b=3.25) where MLE $\hat{a} = 35.47,\ \hat{b} = 3.31$

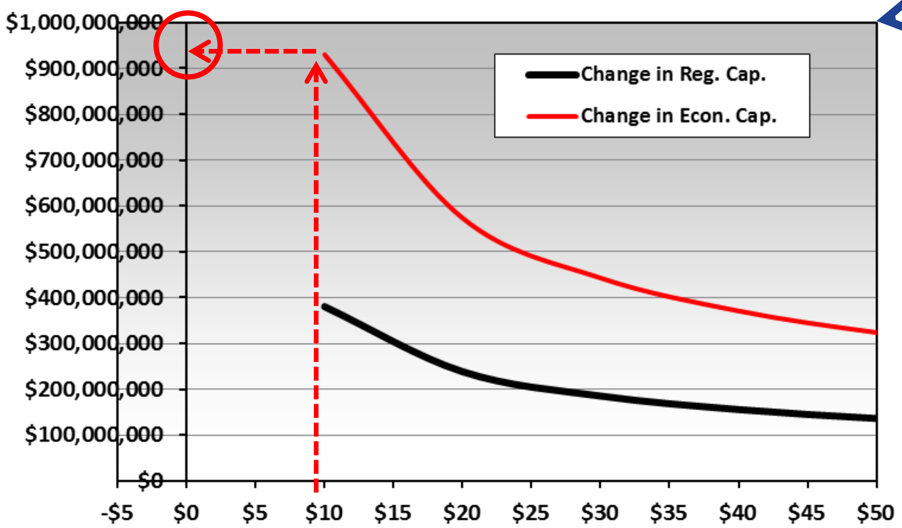


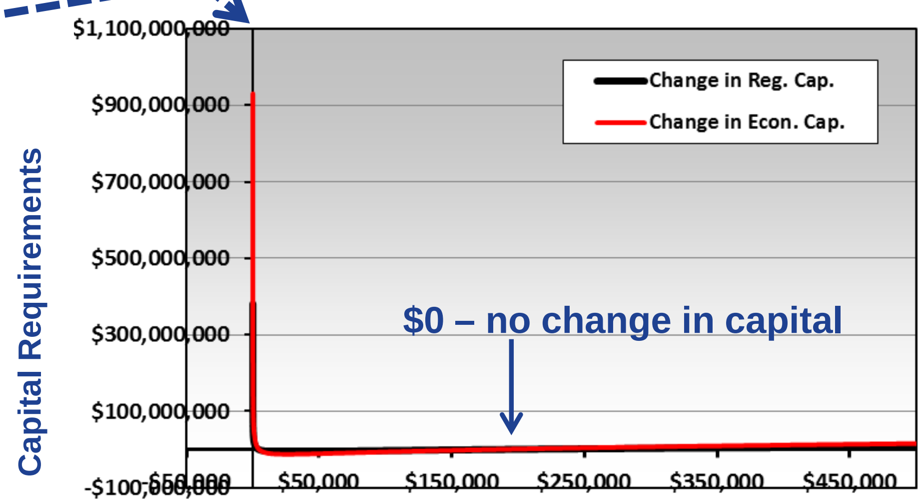
III. Variability of Capital

For MLE, a new loss of $10 increases regulatory capital by over $380m, and economic capital by over $930m. But a loss of about $175k increases capital by $0.



N=250, λ = 25, regulatory α = 0.999, economic α = 0.9997



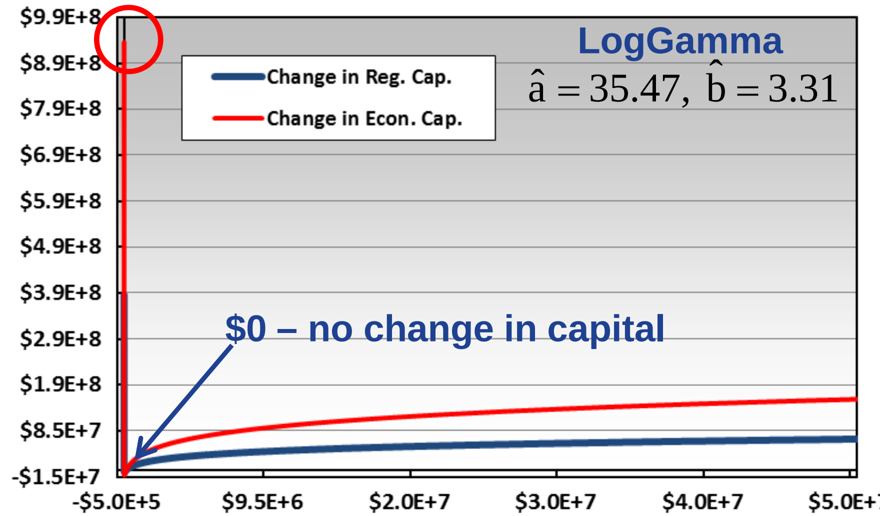


III. Variability of Capital

WHY? Check the MLE IF, which we derived previously as:

$$IF_{\theta}\left(x;\theta,T\right)=$$

$$=\left[\begin{array}{c}\dfrac{\dfrac{a}{b^{2}}\left[\ln(b)+\ln\left(\ln(x)\right)-digamma(a)\right]-\dfrac{1}{b}\left[\ln(x)-\dfrac{a}{b}\right]}{trigamma(a)\left(\dfrac{a}{b^{2}}\right)-\dfrac{1}{b^{2}}}\\[2ex]\dfrac{\dfrac{1}{b}\left[\ln(b)+\ln\left(\ln(x)\right)-digamma(a)\right]-trigamma(a)\left[\ln(x)-\dfrac{a}{b}\right]}{trigamma(a)\left(\dfrac{a}{b^{2}}\right)-\dfrac{1}{b^{2}}}\end{array}\right]$$



N=250, λ = 25, regulatory α = 0.999, economic α = 0.9997

Here, ln(x) in BOTH IF terms dominate the ln(ln(x)) terms, so ln(ln(x)) – ln(x), which attains a global maximum at x=exp(1), becomes a large negative number as $x \rightarrow 1^{+}$. However, for the LogGamma smaller b uniformly INCREASES the quantiles of the distribution, while smaller a DECREASES them. The b term dominates, however, because of the relative size of the constants in both numerators, so capital increases without bound as $x \rightarrow 1^{+}$.

III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

   d. **High-Probability, Low-Severity Loss Events**
   (Yes, you read this right)

MYTH: Low Probability, High Severity losses drive the excessive variability of the capital estimate under the loss distribution approach.

REALITY: This is not true per se. MOST of the excessive quarter-to-quarter variability in capital estimation is due to High-Probability, Low-Severity Loss Events near the data collection threshold. Low-Probability, High-Severity losses ALSO contribute to excessive variability in the capital estimate, but they are so rare that they do not make up the lion's share of quarter-to-quarter variability. In other words, maximum losses (or close) will drive the capital estimate WHEN THEY HAPPEN, but they do not make it bounce around so much from quarter-to-quarter, often when risk profiles do not change at all!

III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

   e. **“behavioral convexity” of VaR as a function of severity parameter estimates** (presented in the next section on Inflation (Size) of Capital).

III. Variability of Capital

iii. This EXCESSIVE VARIABILITY is a function of five things:

   f. **Systemically Upward Capital Bias (inflation) due to an essential discontinuity** in the most commonly used quantile approximation: Degen's (2010) Single Loss Approximation (SLA).
   (presented in the next section on Inflation (Size) of Capital).

IV. Size of Capital

Inflated/Large Size of the Operational Risk Capital Estimate

i. The Inflated/Large Size of Estimated OpRisk Capital is a function of five things:

"old"

a. the EXCESSIVELY high severity percentile that must be estimated based on the 99.9%tile of the Aggregate Loss Distribution
b. All relevant severities are heavy-tailed

"new"

c. Systematically Upward Capital Bias (inflation) due to "Behavioral Convexity" of VaR as a function of severity parameter estimates
d. Both Low-Probability, High-Severity events, as well as SOME High-Probability, Low-Severity events
e. Systemically Upward Capital Bias (inflation) due to an essential discontinuity in the most commonly used quantile approximation: Degen's (2010) Single Loss Approximation (SLA)

IV. Size of Capital

i. Inflated/Large Size of Estimated OpRisk Capital is a function of five things

   a. **the EXCESSIVELY high severity percentile** that must be estimated based on the 99.9%tile of the Aggregate Loss Distribution

   As shown above, a 99.9%tile of the ALD corresponds to estimating a much higher (sometimes 99.999%tile+) of the severity distribution, which corresponds to an extremely large quantile ($ amount).

   Despite persistent efforts from the industry, this regulatory requirement has not changed in over a decade, and most expect that it will not change.

   Developing a systematic, robust method for accurately “scaling” this high severity quantile has proven very challenging, and this problem has not yet been solved.

   Other metrics, such as CVaR, whose use is debated for other risk types, would yield even HIGHER capital estimates in this setting, especially because Operational Risk deals with such heavy-tailed severity distributions.

IV. Size of Capital

i. Inflated/Large Size of Estimated OpRisk Capital is a function of five things

b. All relevant severities are Heavy-Tailed

Severity distributions that are not at least fairly heavy-tailed are not going to pass regulatory muster.

When estimating the same high %tiles, those of heavier-tailed severity distributions will have larger quantiles ($ amounts) compared to those of lighter-tailed distributions.

IV. Size of Capital

i. Inflated/Large Size of Estimated OpRisk Capital is a function of five things

c. <u>**Systematically Upward Capital Bias**</u> (inflation) due to Convexity of VaR as a function of severity parameter estimates

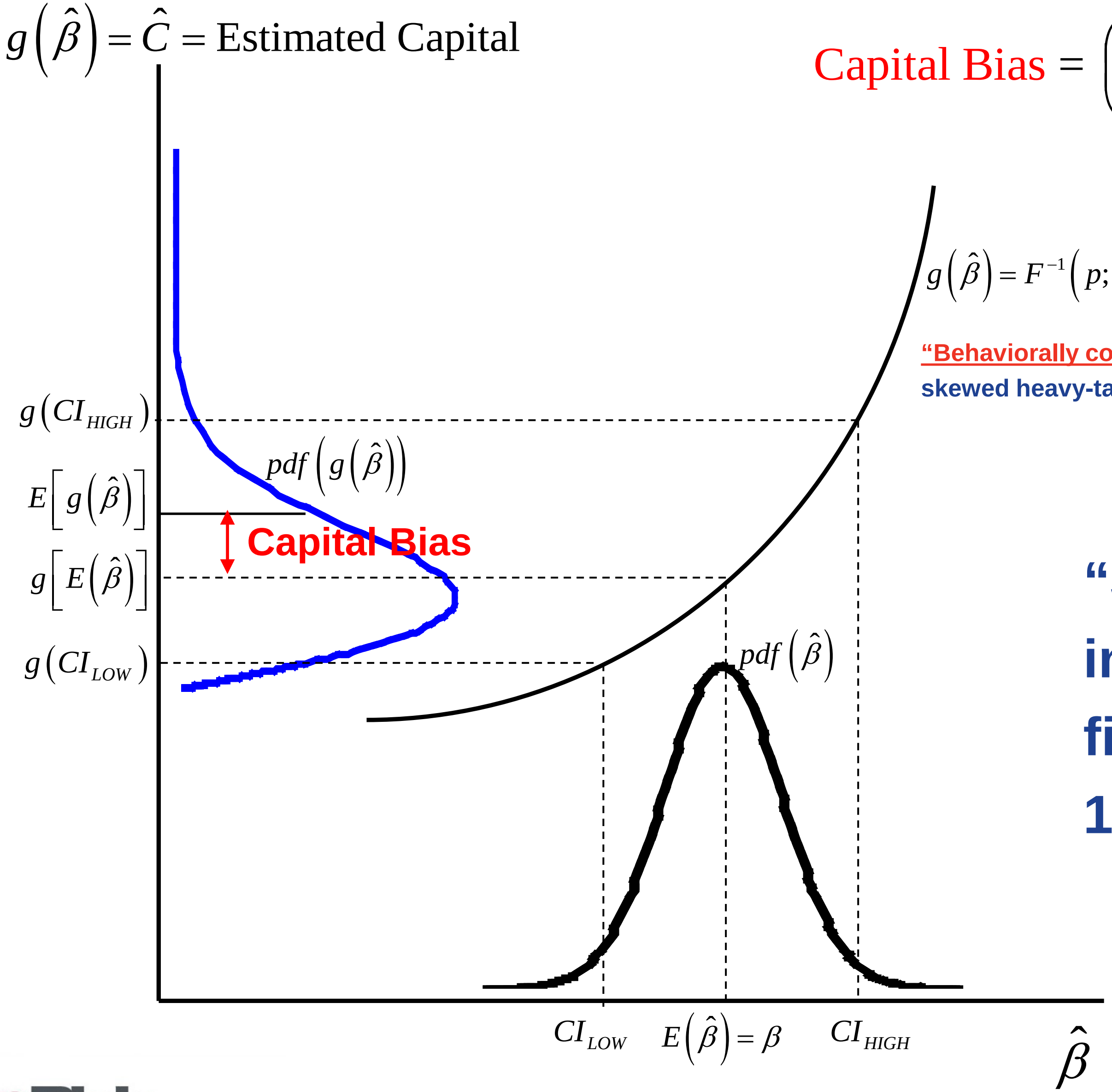
Operational Risk Capital Estimated under the Loss Distribution Approach is systematically inflated (i.e. biased upwards). This inflation is material much, if not most of the time, especially for larger financial institutions (see Opdyke, 2014; Ergashev et al., 2014; RMA, 2013; and Opdyke and Cavallo, 2012a, 2012b).

Unbiased severity parameter estimation does not change this result, and even robust severity parameter estimation does not substantially mitigate this result. Simply put, the choice of estimator does not make much difference.

Because this inflation can be extremely large in both absolute and relative terms (into the billions of dollars, and sometimes more than doubling the estimate over "true" capital), it also strongly contributes to the Excessive Variability of the Capital Estimate.

IV. Size of Capital

$$\text{Capital Bias} = \left(E\left[g\left(\hat{\beta}\right) \right] - g\left[E\left(\hat{\beta}\right) \right] \right) > 0$$



"Behaviorally convex" for the extreme tail of OpRisk-relevant, skewed heavy-tailed severities.

"Jensen's inequality" first proved in 1906.

Graph based on Kennedy (1992), p.37.

IV. Size of Capital

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RCE vs. LDA-MLE for Truncated LogNormal Severity (μ = 10.7, σ = 2.385, H=$10k)*
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IV. Size of Capital

Table 2:
RCE vs. LDA-MLE for GPD Severity (ξ = 0.875, θ = 47,500, H=$0k)*
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IV. Size of Capital

i. Inflated/Large Size of Estimated OpRisk Capital is a function of five things

  c. **Systematically Upward Capital Bias** (inflation) due to Convexity of VaR as a function of severity parameter estimates

MYTH: If our severity parameter estimates are unbiased, our capital is unbiased.

REALITY: This is DEFINITELY not true, and shows how extensive research into parameter estimation can be better directed toward UNBIASED CAPITAL ESTIMATION.

MYTH: Choosing the right severity estimator will mitigate or eliminate this problem.

REALITY: No widely used estimator can substantially mitigate this capital bias.

IV. Size of Capital

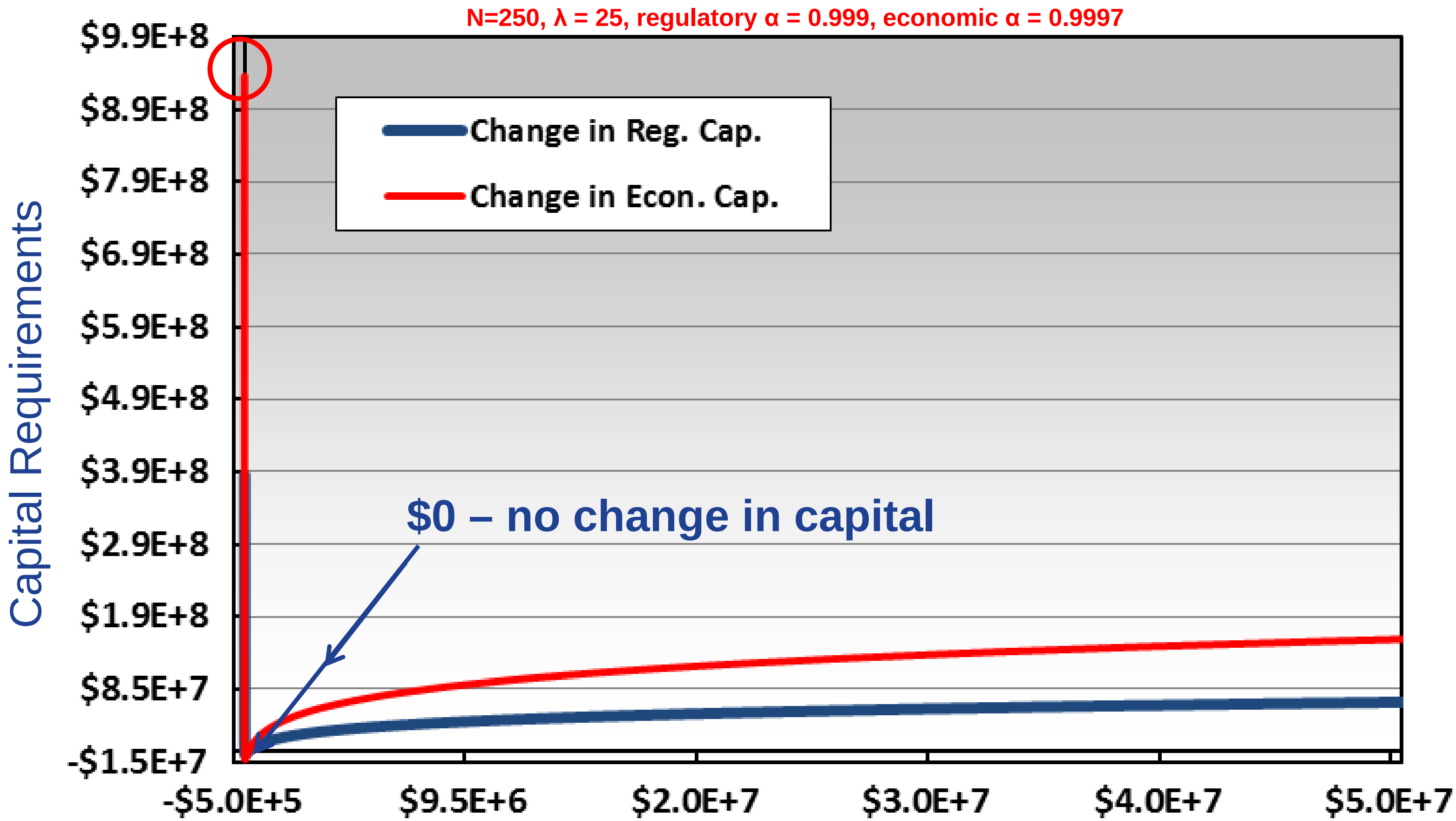
i. Inflated/Large Size of Estimated OpRisk Capital is a function of five things

   d. **<u>Both Low-Probability, High-Severity events, as well as SOME High- Probability, Low-Severity events</u>**

   As described above, MOST of the excessive quarter-to-quarter variability in capital estimation is due to High-Probability, Low-Severity Loss Events near the data collection threshold (DCT). These losses, even if only modestly close to the DCT, disproportionately and often dramatically increase the SIZE of estimated capital, as shown on the Influence Function slides above. This result is counterintuitive, but the math is irrefutable, so we must remain cognizant of this "blind spot" / weakness of the loss distribution approach framework (see below for one published method that directly and at least partially addresses this problem).

IV. Size of Capital

Based on a Random Draw from LogGamma (a=35.5, b=3.25) where MLE $\hat{a} = 35.47, \ \hat{b} = 3.31$



IV. Size of Capital

i. Inflated/Large Size of Estimated OpRisk Capital is a function of five things

   d. <u>Both Low-Probability, High-Severity events, as well as SOME High- Probability, Low-Severity events</u>

   Of course, WHEN THEY HAPPEN, Low-Probability, High-Severity losses will drive the capital estimate. This is obviously most true of maximum losses, which depending on the severity, can lead to the "one loss causes ruin" problem.

   But it is critical to note the relative frequency of HPLS vs. LPHS losses: the former can cause just as much damage, and can do so far more frequently. So **perhaps much of the hype about LPHS should be redirected towards HPLS losses!**

IV. Size of Capital

i. Inflated/Large Size of Estimated OpRisk Capital is a function of five things

   d. **Both Low-Probability, High-Severity events, as well as SOME High- Probability, Low-Severity events**

MYTH: Low Probability, High Severity losses drive the size of the capital estimate.

REALITY: This is not true per se. High-Probability, Low-Severity loss events near the data collection threshold can cause just as much damage. AND THEY OCCUR FAR MORE OFTEN! So perhaps from a resource-allocation perspective, these should be given more of the attention that has been directed towards extremely rare loss events, at least when capital is estimated under a Loss Distribution Approach framework.

"Fixing" this counterintuitive weakness of the Loss Distribution Approach is one topic addressed in Opdyke (2014).

IV. Size of Capital

i. Inflated/Large Size of Estimated OpRisk Capital is a function of five things

   e. **<u>Systemically Upward Capital Bias (inflation) due to an essential discontinuity</u>** in the most commonly used quantile approximation: Degen's (2010) Single Loss Approximation (SLA).

      Of the ten or so published methods for approximating the 99.9%tile of the Aggregate Loss Distribution, SLA is the most widely used and has the advantage of being a closed-form analytical approximation (in other words, it is a formula, not a time-consuming simulation).

      However, even when infinite mean severities are excluded from consideration (which is by no means the default choice), SLA can often generate systematically upward biased estimates due to an essential discontinuity in its equation (see below).

IV. Size of Capital

- **Under the Basel II/III AMA, estimated capital requirements are the Value-at-Risk (VaR) quantile corresponding to the 99.9%Tile of the aggregate loss distribution, which is the convolution of the frequency and severity distributions. This convolution typically has no closed form, but its VaR may be obtained in a number of ways, including extensive monte carlo simulations, fast Fourier transform, Panjer recursion (see Panjer (2006) and Embrechts and Frei (2009)), and Degen's (2010) Single Loss Approximation. All are approximations, with the first as the gold standard providing arbitrary precision, and SLA as the fastest and most computationally efficient. SLA is implemented as below under three conditions (only a) is relevant for severities that cannot have infinite mean):**

a) if $\xi < 1,$ $$C_\alpha \approx F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)+\lambda\mu$$ **where** μ **is the mean of** F **and** ξ **is its tail index**

b) if $\xi = 1,$ $$C_\alpha \approx F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)+c_\xi\lambda\mu_F\left[F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)\right]$$ **where** $c_\xi = 1,\ \mu_F(x)=\int_0^x\left[1-F(s)\right]ds$

c) if $1<\xi<2,$ $$C_\alpha \approx F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)-(1-\alpha)F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)\cdot\left(\frac{c_\xi}{1-1/\xi}\right)$$ **where** $c_\xi = (1-\xi)\dfrac{\Gamma^2(1-1/\xi)}{2\Gamma(1-2/\xi)}$

($\xi \geq 2$ is so extreme as to not be relevant in this setting)

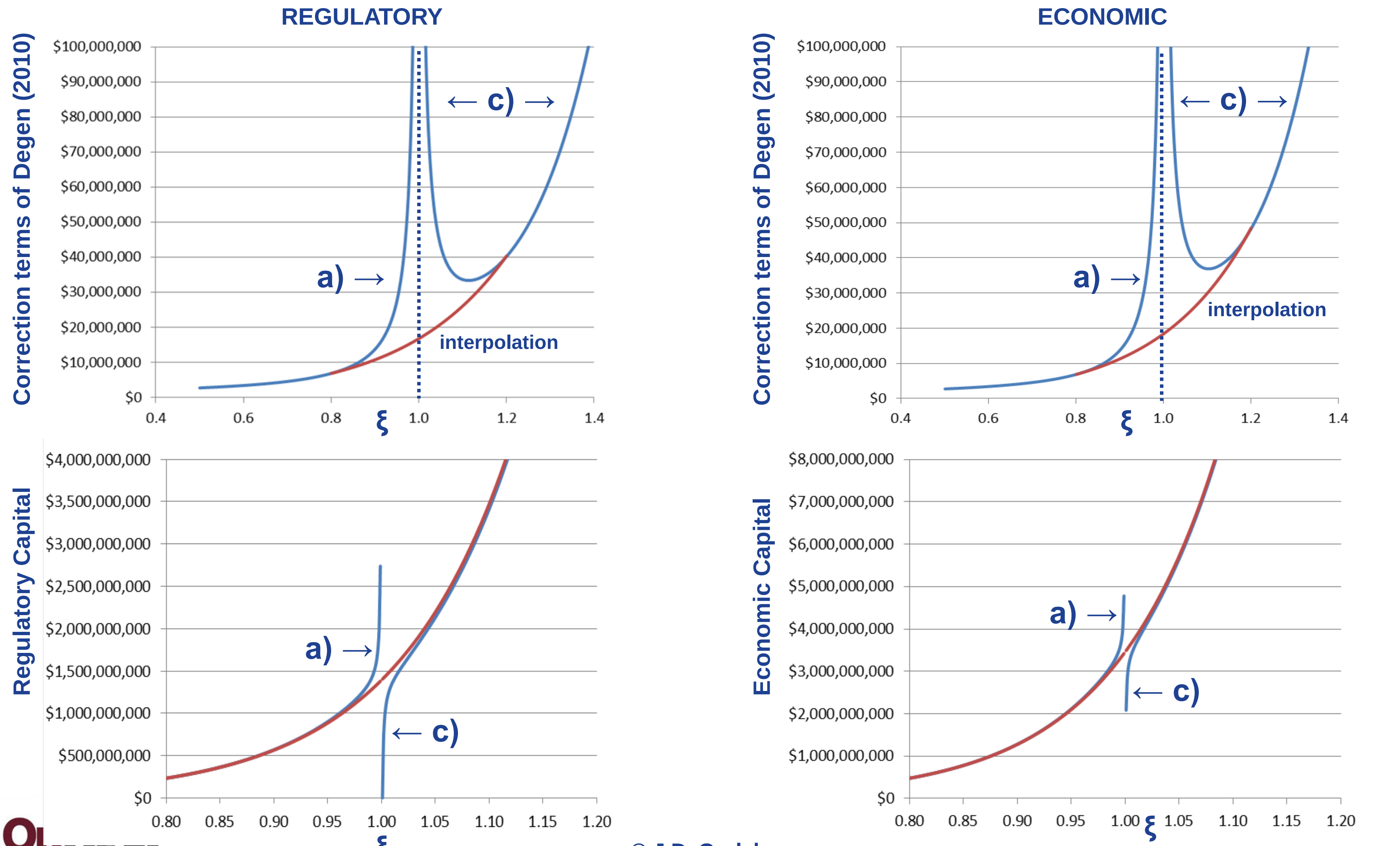
(the above assumes a Poisson-distributed frequency distribution and can be modified if this assumption does not hold)

- **When implementing the above it is important to note that the capital estimate diverges as $\xi \to 1$; specifically, for a) $C_\alpha \to +\infty$ as $\xi \to 1^-$ and for c) $C_\alpha \to -\infty$ as $\xi \to 1^+$. Note that this divergence does not only occur for small deviations from $\xi = 1$. For example, for GPD, divergence can be noticeable in the range of $0.8 < \xi < 1.2$. Therefore, one must utilize a nonlinear interpolation or an alternative derivation of Degen's formulae to avoid this obstacle. All results relying on SLA herein utilize the former solution – i.e. "ISLA" (see Opdyke, 2014) and were all tested to be within 0.5% of extensive Monte Carlo results (e.g. 100 million years' worth of Monte Carlo loss simulations).**

IV. Size of Capital

OpRisk NorthAmerica

Figures A1-A4: ISLA Correction for SLA Discontinuity at ξ=1 for GPD Severity (θ = 55,000)



OpRisk NorthAmerica

IV. Size of Capital

i. Inflated/Large Size of Estimated OpRisk Capital is a function of five things

   e. **<u>Systemically Upward Capital Bias (inflation) due to an essential discontinuity</u>** in the most commonly used quantile approximation: Degen's (2010) Single Loss Approximation (SLA).

      The capital inflation that can result from the discontinuity in Degen's (2010) SLA can be very large in absolute terms (hundreds of millions of dollars).

      This also obviously affects capital variability, in some cases dramatically if the tail index is close to a value of one.

      The Interpolated Single Loss Approximation (ISLA) of Opdyke (2014) is a very straightforward "fix" that solves this problem without sacrificing the speed that is the greatest benefit of using SLA over other approaches.

IV. Size of Capital

i. Inflated/Large Size of Estimated OpRisk Capital is a function of five things

e. **<u>Systemically Upward Capital Bias (inflation) due to an essential discontinuity</u>** in the most commonly used quantile approximation: Degen’s (2010) Single Loss Approximation (SLA).

<u>MYTH</u>: Degen’s (2010) Single-Loss Approximation provides an unbiased approximation of the high quantile of the Aggregate Loss Distribution.

<u>REALITY</u>: This is not true when the tail index (of severities that CAN have infinite mean) approaches 1.0, in which case it is systematically biased upwards (i.e. capital is inflated, materially) due to a discontinuity in the formula. Opdyke (2014) provides a “fix” to this issue with Interpolated SLA (ISLA).

V. Choice of Severity Estimator

The Choice of the Severity Estimator, while important, CANNOT solve either of the two main challenges of estimating Operational Risk Capital Under the Loss Distribution Approach: **Excessive Capital Variability and Inflated Capital**

i. Many different severity estimators have been brought to bear in this setting, including but not limited to:
   a. Maximum likelihood estimation (MLE; see Opdyke and Cavallo, 2012a,2012b)
   b. Penalized likelihood estimation (PLE; see Cope, 2011)
   c. Method of Moments (see Dutta and Perry, 2007)
   d. Generalized Method of Moments (see RMA, 2013)
   e. Probability Weighted Moments (PWM; see BIS, 2011)
   f. Bayesian Estimators (see Zhou et al., 2013)
   g. Extreme Value Theory-Peaks Over Threshold (EVT-POT; see Chavez-Demoulin et al., 2014)
   h. Quantile Distance Estimator (QD; see Ergashev, 2008)
   i. Optimal Bias-Robust Estimator (OBRE; see Opdyke and Cavallo, 2012a)
   j. Cramer von Mises Estimator (CvM, not to be confused with the goodness-of-fit statistic of the same name; see Opdyke and Cavallo, 2012a)
   k. Generalized Median Estimator (see Serfling, 2002; Wilde and Grimshaw, 2013)
   l. Probability Integral Transform statistic (PITS; see Finkelsteign et al., 2006)

V. Choice of Severity Estimator

The Choice of the Severity Estimator, while important, CANNOT solve either of the two main challenges of estimating Operational Risk Capital Under the Loss Distribution Approach: **Excessive Capital Variability and Inflated Capital**

ii. **None of the severity parameter estimators can effectively address the issue of inflated capital**, because unbiased parameter estimation is NOT the issue here: AFTER severity parameters are estimated, VaR (the quantile of the 99.9%tile) of the Aggregate Loss Distribution is obtained, and VaR is a "behaviorally convex" function* of the severity parameters, as shown previously.

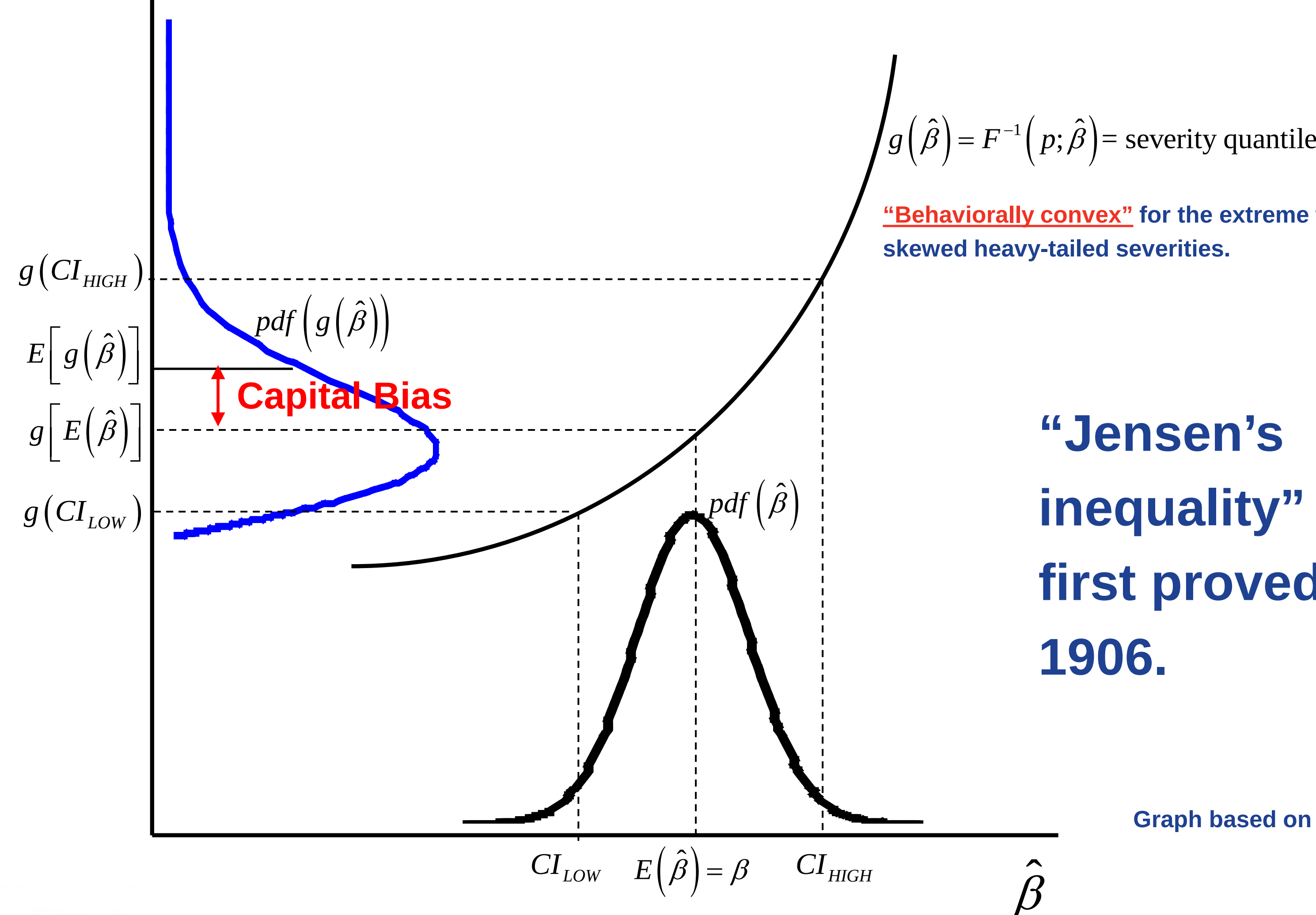
So no matter how "on target" the severity parameter – it could be a perfect bullseye on average! – capital STILL will be inflated. And multiple papers (see Opdyke and Cavallo, 2012a; and Joris, 2013) have shown that increasing parameter robustness significantly (say, via the use of OBRE instead of MLE) has relatively little impact on this capital inflation.

*NOTE: Preliminary research suggests that even when the multivariate VaR surface (as a function of multiple severity parameters) is not strictly convex, it is "behaviorally convex" so that the net effect empirically, when severity parameters vary as dictated by their dependence structure, is that Jensen's Inequality holds for capital (VaR) as a function of the vector of severity parameters.

V. Choice of Severity Estimator

$g\left(\hat{\beta}\right) = \hat{C} = \text{Estimated Capital}$

$$\text{Capital Bias} = \left(E\left[g\left(\hat{\beta}\right)\right] - g\left[E\left(\hat{\beta}\right)\right]\right) > 0$$



"Behaviorally convex" for the extreme tail of OpRisk-relevant, skewed heavy-tailed severities.

"Jensen's inequality" first proved in 1906.

Graph based on Kennedy (1992), p.37.

V. Choice of Severity Estimator

The Choice of the Severity Estimator, while important, CANNOT solve either of the two main challenges of estimating Operational Risk Capital Under the Loss Distribution Approach: **Excessive Capital Variability and Inflated Capital**

iii. **Also, none of the severity parameter estimators can effectively address the issue of excessive capital variability**, simply because the VaR (quantile) of the ALD being estimated is so high (and this translates into an even HIGHER quantile of the severity distribution).

Intuitively, recall the 2x6: the board is simply too long for there NOT to be an enormous "wobble" when we shake it. In other words, when we sample from the data generating mechanism (shake the board), the far end of the board is just going to bounce around too much to get a precise marking on it: it'll change dramatically (vary a lot) every time we draw a different sample! This will hold true even when we have tens of thousands of operational risk loss events, and even when those losses are perfectly i.i.d., because the %tile we are trying to estimate is so high. This is what many statisticians would call an ill-posed problem.

VI. EVT Models

Reliance on Extreme Value Theory (EVT) per se does not solve either of the two main challenges, so the use of EVT models per se will not help under a loss distribution approach framework.

i. **EVT Simplified:** Taking EVT-POT (Peaks-over-Threshold), the piece of EVT relevant to this setting holds that for i.i.d. loss event data, beyond a certain high threshold the distribution of the severity tail converges asymptotically (as sample size approaches infinity) to a Generalized Pareto Distribution (GPD). This is true of all severities relevant to this setting. The quality of this convergence increases/decreases as the threshold is increased/decreased.

ii. An entire literature exists on estimators of the tail index of the GPD, the most common and established being the Hill (1975) and Pickands (1975) estimators (numerous modifications of each exist, some of which eliminate finite sample bias).

VI. EVT Models

Reliance on Extreme Value Theory (EVT) per se does not solve either of the two main challenges, so the use of EVT models per se will not help under a loss distribution approach framework.

iii. The above is an important theoretical result, but its practical application remains challenging for a number of reasons, the foremost being that **the higher the threshold and the better the GPD approximation, the fewer the observations we have to get a good estimate of the tail index (i.e. the key severity distribution parameter)! This is a difficult catch 22.**

iv. This is a challenging and open statistical problem for which the literature provides many stylized and partial solutions, among them a recent one from Miranda (2014) in the Journal of Operational Risk. However, even if a solution to this problem was widely accepted and stood the test of time (and none has), this does not solve the 2 challenges of excessive capital variability and inflated capital size. EVT models do not appear to address this any better than severity models that make use of the entire severity distribution of loss events, rather than just the tail.

VII. What's an OpRisk Modeler to Do?

- There are numerous nontrivial challenges to estimating capital under a loss distribution approach with reasonable accuracy, reasonable precision (i.e. reasonably small variability), and reasonable robustness (i.e. robust under violations of assumptions like i.i.d. data, which are widely recognized as being routinely violated in this setting). Only SOME of these are listed above.
- So what are operational risk modelers to do?

VII. What's an OpRisk Modeler to Do?

- First, we cannot tackle all the abovementioned issues if we do not correctly and objectively define them and face them head-on, even if some appear insurmountable and/or challenge current assumptions about widely used frameworks (e.g. High-Probability, Low Severity losses near the threshold can dramatically INCREASE capital!). This is risky, and easier said than done.
- Secondly, we must design solutions that focus on these now correctly and objectively defined and material problems. This, too, is risky (no low-hanging research fruit), and easier said than done. For example, it is easier to make marginal contributions to the already extensive literature on severity parameter estimation than it is to tackle the very hard problem of convexity in VaR as a function of the severity parameters.
- Only when we accurately and honestly define the problems, and then focus on them by directing resources toward what is admittedly higher-risk applied research (i.e. some of which may be unsolvable or at least non-monetizable), will we have a reasonable chance of making real headway.

VII. One Published Method

Only one published method directly addresses both excessive capital variability and inflated capital size under the loss distribution approach: the Reduced-Bias Capital Estimator (RCE) of Opdyke (2014).

RCE is an estimator of CAPITAL, not of severity parameters, under the loss distribution approach. RCE

i. Nearly eliminates capital inflation due to VaR's "behavioral convexity" as a function of severity parameters.

ii. Dramatically mitigates excessive capital variability (by any measure of spread), very often with decreases of over 50%.

iii. Mitigates the counterintuitive increases in capital that result from High-Probability, Low-Severity losses near the data collection threshold.

Opdyke (2014) also presents an accurate method for approximating the high quantile of the Aggregate Loss Distribution – ISLA – that circumvents the capital-biasing discontinuity of Degen's (2010) SLA … WITHOUT losing the computational speed advantage of using a closed-form analytical formula.

VII. One Published Method

Opdyke's (2014) RCE does not directly "solve" capital variability caused strictly by the extreme size of the quantile that must be estimated, but it does mitigate variability in the capital estimate by mitigating the effects of VaR's "behavioral convexity," which are considerable as shown below in Tables 1 and 2.

VII. One Published Method

Table 1:
RCE vs. LDA-MLE for Truncated LogNormal Severity (μ = 10.7, σ = 2.385, H=$10k)*
[see Opdyke (2014) for complete results]

(millions)	Regulatory Capital**			Economic Capital**	
	RCE	LDA-MLE		RCE	LDA-MLE
Mean*	$700	$847		$1,338	$1,678
True Capital	$670	$670		$1,267	$1,267
Bias (Mean - True)	$30	$177		$71	$411
Bias %	4.5%	26.4%		5.6%	32.4%
RMSE*	$469	$665		$1,003	$1,521
STDDev*	$468	$641		$1,000	$1,464

* 1,000 Simulations, n ≈ 250

** λ = 25; α = 0.999 RC; α = 0.9997 EC

VII. One Published Method

Table 2:
RCE vs. LDA-MLE for GPD Severity (ξ = 0.875, θ = 47,500, H=$0k)*
[see Opdyke (2014) for complete results]

(millions)	Regulatory Capital**			Economic Capital**	
	RCE	LDA-MLE		RCE	LDA-MLE
Mean*	$396	$640		$1,016	$2,123
True Capital	$391	$391		$1,106	$1,106
Bias (Mean - True)	$5	$249		$24	$1,016
Bias %	1.2%	63.7%		2.2%	91.9%
RMSE*	$466	$870		$1,594	$3,514
STDDev*	$466	$834		$1,594	$3,363

* 1,000 Simulations, n ≈ 250

** λ = 25; α = 0.999 RC; α = 0.9997 EC

VII. One Published Method

Other benefits of RCE include:

a. RCE is consistent with the loss distribution approach framework (even with new guidance (6/30/14) encouraging new methods, arguably the smaller the divergence from widespread industry practice, the greater the chances of regulatory approval).
b. RCE works across a very wide range of very different severities.
c. RCE works when severity distributions are truncated to account for data collection thresholds.
d. RCE works even under infinite mean (or close, which is relevant for any simulation-based method even if infinite means are excluded).
e. RCE is not computationally intensive (it can be implemented on a desktop computer).
f. RCE's range of application encompasses all commonly used estimators of severity (and frequency)
g. RCE works regardless of the method used to approximate VaR of the aggregate loss distribution.
h. RCE is easily understood and implemented using any widely available statistical software.
i. RCE provides unambiguous, material improvements over the most widely used implementations of the loss distribution approach (e.g. MLE, and most other estimators) on all three key criteria – capital accuracy, capital precision, and capital robustness.

VIII. Summary and Conclusions

- The loss distribution approach under Basel II's AMA at first blush seems like a relatively straightforward actuarial approach to operational risk capital estimation. But it quickly becomes very complex under the empirical, methodological, and regulatory constraints imposed in this setting.
- These constraints usually interact in nontrivial and complicated ways.
- OpRisk Capital Estimation Research to date has not directly focused on its 2 main challenges: Excessive Capital Variability, & Inflated Capital Size.
- The only way to either solve these problems, circumvent them, or prove that some aspects of them cannot be solved, is to first recognize these as the OpRisk Capital's biggest, most material problems; and then to focus our analytical lenses directly on them rather than on marginal, related issues that are not really driving capital estimation (e.g. severity estimation).
- Opdyke (2014) is the only published paper that directly addresses both excessive capital variability and inflated capital size under the loss distribution approach with the Reduced-Bias Capital Estimator (RCE).
- Further research focused directly on these two challenges is absolutely necessary if the LDA is to be at all useful in this setting, and if large, regulated financial institutions are to be able to generate OpRisk Capital Estimates that are reasonably accurate, precise, and robust.

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J.D. Opdyke
Head of Operational Risk Modeling
GE Capital

J.D.Opdyke@ge.com