Estimating Operational Risk Capital with Greater Accuracy, Precision, and Robustness

J.D. Opdyke[1]
Head of Operational Risk Modeling, Quantitative Methods Group, GE Capital

Forthcoming, *Journal of Operational Risk*, Issue 9, No. 4, December, 2014.

The largest US banks and Systemically Important Financial Institutions are required by regulatory mandate to estimate the operational risk capital they must hold using an Advanced Measurement Approach (AMA) as defined by the Basel II/III Accords. Most of these institutions use the Loss Distribution Approach (LDA) which defines the aggregate loss distribution as the convolution of a frequency distribution and a severity distribution representing the number and magnitude of losses, respectively. Capital is a Value-at-Risk estimate of this annual loss distribution (i.e. the quantile corresponding to the 99.9%tile, representing a one-in-a-thousand-year loss, on average). In practice, the severity distribution drives the capital estimate, which is essentially a very large quantile of the estimated severity distribution. Unfortunately, when using LDA with any of the widely used severity distributions (i.e. heavy-tailed, skewed distributions), all unbiased estimators of severity distribution parameters appear to generate biased capital estimates due to Jensen's Inequality: VaR always appears to be a convex function of these severities' parameter estimates because the (severity) quantile being estimated is so large *and* the severities are heavy-tailed. The resulting bias means that capital requirements always will be overstated, and this inflation is sometimes enormous (sometimes even billions of dollars at the unit-of-measure level). Herein I present an estimator of capital that essentially eliminates this upward bias when used with any commonly used severity parameter estimator. The Reduced-bias Capital Estimator (RCE), consequently, is more consistent with regulatory intent regarding the responsible implementation of the LDA framework than other implementations that fail to mitigate, if not eliminate this bias. RCE also notably increases the precision of the capital estimate and consistently increases its robustness to violations of the i.i.d. data presumption (which are endemic to operational risk loss event data). So with greater capital accuracy, precision, and robustness, RCE lowers capital requirements at both the unit-of-measure and enterprise levels, increases capital stability from quarter to quarter, ceteris paribus, and does both while more accurately and precisely reflecting regulatory intent. RCE is straightforward to explain, understand, and implement using any major statistical software package.

Keywords: Operational Risk, Basel II, Jensen's Inequality, AMA, LDA, Regulatory Capital, Economic Capital, Severity Distribution

---

[1] J.D. Opdyke is Head of Operational Risk Modeling, Quantitative Methods Group at GE Capital where he is heading up all operational risk modeling and quantification efforts including AMA capital estimation, CCAR, and all related modeling (e.g. ECap) and reporting. J.D. has over 20 years of experience as a quantitative consultant, most of this in the banking and credit sectors where his clients have included multiple Fortune and Global 50 banks and financial credit organizations. J.D.'s recent Journal of Operational Risk paper (2012) was voted "Paper of the Year" by Operational Risk & Regulation staff in consultation with industry experts, and he has been invited to present his work at the American Bankers Association Operational Risk Forum, the Operational Risk Exchange Analytics Forum, and OpRisk North America 2014. J.D.'s other publications span statistical finance, computational statistics (solving "Big Data" problems using SAS®), number theory/combinatorics, and applied econometrics. J.D. earned his undergraduate degree, with honors, from Yale University, his Master's degree from Harvard University where he was awarded both Kennedy and Social Policy Research Fellowships, and he completed post-graduate statistics work as an ASP Fellow in the graduate mathematics department at MIT.

The views expressed in this paper are the views of the sole author and are not necessarily those of GE Capital or any other institution. The author may be contacted at J.D.Opdyke@GE.com.

The author extends his sincere appreciation to Toyo R. Johnson, Nicole Opdyke, and Ryan Opdyke for their thoughtful insights, and to Kirill Mayorov for a thorough review and for spotting an error in an earlier draft.

"Measurement is the first step that leads to control and eventually to improvement. If you can't measure something, you can't understand it. If you can't understand it, you can't control it. If you can't control it, you can't improve it." - H. J. Harrington

Background, Introduction, and Objectives

In the United States, regulatory mandate is compelling the larger banks[2] and companies designated as Systemically Important Financial Institutions ("SIFIs," both bank and non-bank)[3] to use an Advanced Measurement Approach (AMA) framework to estimate the operational risk capital they must hold in reserve.[4] Both industry practice and regulatory guidance have converged over the past decade[5] on the Loss Distribution Approach (LDA)[6] as the most widely used AMA framework. Under this approach, data on operational risk loss events[7] is used to estimate a frequency distribution, representing the *number* of loss events that could occur over a given time period (typically a year), and to estimate a severity distribution, representing the *magnitude* of those loss events. These two distributions are then combined via convolution to obtain an annual aggregate loss distribution. Operational risk regulatory capital (RCap) is the dollar amount associated with the 99.9%tile of this estimated loss distribution. Operational risk economic capital (ECap) is the quantile associated with, typically, the 99.97%tile of the aggregate loss distribution, depending on the institution's credit rating.[8]

The frequency, severity, and capital estimations take place at the level of the Unit-of-Measure (UoM). UoM's simply are the groups into which operational risk loss events are categorized, generally under the competing

[2] These include banks and systemically important financial institutions ("SIFIs") with over $250 billion in total consolidated assets, or over $10 billion in total on-balance sheet foreign exposure (and includes the depository institution subsidiaries of these firms). See Federal Register (2007).

[3] On July 8, 2013, the Financial Stability Oversight Council of the U.S. Department of the Treasury, as authorized by Section 113 of the Dodd-Frank Act, voted to designate American International Group (AIG) and General Electric Capital Corporation, Inc. (GECC) as SIFIs. On September 19, 2013, the Council voted to designate Prudential Financial, Inc. a SIFI. See www.treasury.gov/initiatives/fsoc/designations/Pages/default.aspx

[4] See BCBS (2004). The other two, less empirically sophisticated methods – the Basic Indicator Approach and the Standardized Approach – are simple functions of gross income. As such, they are not risk sensitive and do not accurately reflect the complex risk profiles of these financial institutions.

[5] There have been no dramatic changes with respect to operational risk capital estimation under an AMA since the first comprehensive guidance was published in 2004 (see BCBS, 2004).

[6] This approach has a longer history of use within the insurance industry.

[7] An operational risk loss event only can result from an operational risk, which is defined by Basel II as, "the risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events. This includes legal risk, but excludes strategic and reputational risk." See BCBS (2004).

[8] ECap is higher than RCap as it addresses the very solvency of the institution. The 99.97%tile is a typical value used for ECap (almost all are 99.95%tile or above), based on a firm's credit rating, since it reflects 100% minus the historical likelihood of an AA rated firm defaulting in a one-year period. See Hull (2012).

objectives of homogeneity and (larger) sample size. Basel II identifies eight business lines and seven event types that together comprise fifty-six UoM's. Individual institutions either use some or all of these UoM's as is, define their UoM's empirically, or use some combination of these two approaches. Capital estimated at the UoM level then must be aggregated to a single estimate at the enterprise level, and under the conservative (and unrealistic) assumption of perfect dependence, capital is simply summed across all UoM's. In reality, however, losses do not occur in perfect lockstep across UoM's no matter how they are defined, and so this imperfect dependence in the occurrence of loss events can be estimated and simulated, typically via copula models.[9] This potentially can provide an enormous diversification benefit to the banks/SIFIs,[10] and along with LDA's risk-sensitive nature generally, is the major 'carrot' that counterbalances the 'stick' that is the regulatory requirement of an AMA implementation. These potential benefits also have been a major motivation for LDA's adoption by many institutions beyond the US. For a more extensive and detailed background on the LDA and its widespread use for operational risk capital estimation, see Opdyke and Cavallo (2012a and 2012b).[11]

As described above, capital under the LDA is based on the convolution of the severity and frequency distributions. However, estimates of severity and frequency are exactly that: merely estimates based only on samples of operational risk loss event data. Their values will change from sample to sample, quarter to quarter, and because they are based directly on these varying estimates, the capital estimates, too, will vary from sample to sample, quarter to quarter. So it is essential to understand how this *distribution* of capital estimates is shaped if we are to attempt to make reliable inferences (about "true" capital numbers) based on it. Is it centered on "true" capital values (if we test it using known inputs with simulated data), or is it systematically biased? If biased, in what direction – upwards or downwards – and under what conditions is this bias material? Is the capital distribution reasonably precise, or do capital estimates vary so dramatically as to be completely unreliable and little better than a wild guess at what the true capital values really are? Is the distribution

[9] There are other approaches to estimating dependence structures and tail dependence in particular, such as mixture models (see Reshetar, 2008), but many are much newer and not yet tested extensively in practice (for example, see Arakelian and Karlis, 2014, Bernard and Vanduffel, 2014, Dhaene et al., 2013, and Polanski et al., 2013).

[10] See RMA (2011), OR&R (2009), and Haubenstock and Hardin (2003).

[11] A key point here that drives a focus of this paper is the fact that empirically, the severity distribution drives capital much more than does the frequency distribution – typically orders of magnitude more. This is true both from the perspective of the choice of which severity distribution is used vs. the choice of which frequency distribution is used (the latter changes capital very little compared to the former), as well as variance in the values of the severity parameters vs. variance in the values of the frequency parameter(s): a change of a standard deviation of the former typically has an enormous effect on estimated capital in both absolute and relative terms, while the same change in the latter has a much smaller, if not de minimis effect on estimated capital. This is well established in the literature (see Opdyke and Cavallo, 2012a and 2012b, and Ames et al., 2014), and the analytic reasons for this are demonstrated later in this paper. So while stochastic frequency parameter(s) always are and always should be included in operational risk capital estimation and simulation, the severity distribution typically (and rightly) is more of a focus of research on operational risk capital estimation than is the frequency distribution.

reasonably robust to real-world violations of the properties of the loss data assumed by the estimation methods, or do modest deviations from idealized, mathematically convenient textbook assumptions effectively distort the results in material ways, and arguably render them useless? These are questions that only can be answered via scrutiny of the entire *distribution* of capital estimates (say, at least one thousand estimates), as opposed to a few capital numbers that may or may not appear to be "reasonable" based on a few estimates of severity and frequency distribution parameters. And we should be ready for answers that may call into question the conceptual soundness of the LDA framework, or at least the manner in which major components of it are commonly implemented in this setting.[12]

This paper addresses these issues directly by focusing on the capital distribution and what are arguably the three biggest challenges to LDA-based operational risk capital estimation: the fact that even under idealized data assumptions,[13] LDA-based capital estimates are i) systematically inflated (and sometimes grossly inflated by many hundreds of millions of dollars under conditions not uncommon for the largest, and even medium-sized banks),[14] ii) extremely imprecise by any reasonable measure (i.e. they are extremely variable from sample to sample – see Opdyke, 2013, Opdyke and Cavallo, 2012a, Cope et al., 2009, and OR&R, 2014, for more on this topic), and iii) extremely non-robust to violations of the (i.i.d.) data assumptions almost always made when implementing the LDA (and which are universally recognized as unrealistic; see, for example, Opdyke and Cavallo, 2012a, and Horbenko et al., 2011). Yet it is precisely these three factors – capital accuracy, capital precision, and capital robustness – that arguably are the only criteria that matter when assessing the efficacy of an operational risk (or any) capital estimation framework. Indeed, the stated requirement of the US Final Rule on the Advanced Measurement Approaches for Operational Risk (see US Final Rule, 2007, and Interagency Guidance, 2011) is for "credible, transparent, systematic, and verifiable processes that incorporate all four operational risk elements … [that should be combined] in a manner that most effectively enables it [the regulated bank/sifi] to quantify its exposure to operational risk." But can it even be seriously argued that an operational risk capital estimation framework that generates results consistent with i), ii), and iii) above could

[12] One such example is the extremely large size of the quantile of the aggregate loss distribution – that corresponding to the 99.9%tile – that firms are required to estimate. See Degen and Embrechts (2011) and Nešlehová et al. (2006) for more details.

[13] The most sweeping, yet common assumption is that the loss data is "i.i.d." – independent and identically distributed. "Independent" means that the values of losses are unrelated across time periods, and "identically distributed" means that losses are generated from the same data generating process, typically characterized as a parametric statistical distribution (see Opdyke and Cavallo, 2012a, 2012b). The assumption that operational risk loss event data is "i.i.d" is widely recognized as unrealistic and made more for mathematical and statistical convenience than as a reflection of empirical reality (see Opdyke and Cavallo, 2012a, 2012b). The consequences of some violations of this assumption are examined later in this paper.

[14] This has been confirmed by empirical findings in the literature (see Opdyke and Cavallo, 2012a and 2012b, Opdyke, 2013, Joris, 2013, and Ergashev et al., 2014) as well as a recent position paper from AMAG ("AMA Group"), a professional association of major financial institutions subject to AMA requirements (see RMA, 2013, which cites the need for "Techniques to remove or mitigate the systematic overstatement (bias) of capital arising in the context of capital estimation with the LDA methodology").

be deemed "credible"? Or even "verifiable" in the face of excessive variability in capital estimate outcomes? How could one even assess whether i), ii), and/or iii) are true without scrutinizing the distribution of capital estimates that the framework generates under controlled conditions (i.e. under well-specified and extensive loss data simulations)?

Unfortunately, very little operational risk research tackles these three issues head-on through a systematic examination of the entire *distribution* of capital estimates, as opposed to simply presenting several capital estimates almost as an afterthought to an analysis that focuses primarily on severity parameter estimation (a few exceptions include Rozenfeld, 2011, Opdyke and Cavallo, 2012a and 2012b, Opdyke, 2013, Joris, 2013, and Zhou, 2013). However, cause for optimism lies in the fact that a single analytical source appears to account for much, if not most of the deleterious effect of these three issues on capital estimation. What has become known as Jensen's inequality – a time-tested analytical result first proven in 1906 (see Jensen, 1906) – appears to be the sole cause of i), as well as a major contributing factor to ii) and, to a lesser extent, iii). Yet this has been overlooked and virtually unmentioned in the operational risk quantification and capital estimation literature (see Opdyke and Cavallo, 2012a, b, Opdyke, 2013, and Joris, 2013 for the only known exceptions).[15] If a fraction of the effort that has gone into research on severity parameter estimation also is directed at capital estimation, and specifically on defining, confronting, and mitigating the biasing, imprecision, and non-robustness effects apparently caused by Jensen's inequality, then all in this space – practitioners, academics, regulated (and even non-regulated) financial institutions, and regulators – quickly will be much farther along the path toward making the existing LDA framework much more useable and valuable in practice.[16] It has been a decade since Basel II published comprehensive guidance on operational risk capital estimation,[17] and still these three issues remain to dog the industry's efforts at effectively utilizing the LDA framework to provide reasonably accurate, reasonably precise, and reasonably robust capital estimates. So we are long past due for a refocusing of our analytical lenses specifically on the capital distribution and on these three challenges to make some real strides

[15] Of course, Jensen's inequality has long been the subject of applied research in other areas of finance (see Fisher et al., 2009), applied econometrics (see Duan, 1983), and even bias in market risk VaR (see, for example, Liu and Luger, 2006). But proposed solutions to its deleterious effects on estimation have not been extended to operational risk capital, the literature for which has almost completely ignored it (with the exception of Opdyke and Cavallo, 2012a and 2012b, Opdyke, 2013, and Joris, 2013). Although it does not identify Jensen's inequality as the source, RMA (2013) does identify "the systematic overstatement (bias) of capital arising in the context of capital estimation with the LDA methodology," and Ergashev et al. (2014) present extensive empirical results exactly consistent with its effects and with the empirical results shown in this paper.

[16] Here, "useable" and "valuable" are based on assessments of the accuracy, precision, and robustness of the capital estimates that the framework generates. A realistic example, shown later in this paper, makes the point: when true capital is, say, $391m, but 1,000 LDA capital estimates (based on 1,000 i.i.d. simulated samples) average $640m with a standard deviation of over $834m, the framework generating the estimates, due to this large upward bias and gross imprecision, unarguably is not terribly useful or valuable to those needing to make business decisions based on its results. And this is under the most idealized i.i.d. data assumptions which are rarely, if ever, realized in actual practice.

[17] See BCBS (2004).

towards providing measurable, implementable, and impactful solutions to them. The direct financial and risk mitigation stakes for getting these capital numbers "right" (according to these three criteria) are enormously high for individual financial institutions (especially the larger ones), as well as for the industry as a whole, so our best efforts as empirical researchers should require no less than this refocusing, if not complete problem resolution.

To this end, this paper has two main objectives: first, to identify and clearly demonstrate that Jensen's inequality is the most like source of the materially deleterious effects on LDA-based operational risk capital estimation, define the specific conditions under which these effects are material, and make the case for a shift in focus to the distribution of capital estimates, rather than focusing solely on the distribution of the severity parameter estimates. After all, capital estimation, not parameter estimation, is the endgame here. And secondly, to develop and propose a capital estimator – the Reduced-bias capital estimator (RCE) – that tackles all three of the major issues mentioned above – capital accuracy, capital precision, and capital robustness – and unambiguously improves capital estimates by all three criteria when compared to the most widely used implementations of LDA based on maximum likelihood estimation (MLE) (and a wide range of similar estimators). Requirements governing the development and design of RCE include:

- Its use and assumptions must not conflict with those supporting the LDA framework specifically,[18] and it must be entirely consistent with regulatory intent regarding this framework's responsible and prudent implementation generally (I argue below that RCE is *more* consistent with regulatory intent in the context of applying the LDA than most, if not all other known implementations of it).[19]
- It must utilize the same general methodological approach across sometimes very different severity distributions, including those that are truncated to account for data collection thresholds.
- It must "work" regardless of whether the mean of the severity distribution is infinite, or close to infinite.
- Its range of application must encompass most, if not all, of the commonly used estimators of the severity (and frequency) distributions.
- It must "work" regardless of the method used to approximate the VaR of the aggregate loss distribution.

---

[18] This is not to say that research that proposes changing the bounds or parameters of the framework is any less valuable per se, but rather, that this was a conscious choice made to maximize the range of application of the proposed solution (RCE). RCE is designed to be entirely consistent with the LDA framework specifically, and regulatory guidance and expectation generally so that an institution's policy decision to strictly adhere to all aspects of the framework would not preclude usage of RCE. In fact, RCE is arguably *more* consistent with regulatory guidance and expectation than are most, if not all other implementations of LDA, because its capital estimates are not systemically biased upwards: they are, on average, quite literally the expected values for capital, or very close, under the LDA framework (in other words, they are centered on true capital). So capital estimates based on RCE arguably are *most* consistent with regulatory intent regarding the responsible implementation of the LDA framework, as discussed below.

[19] It is important to note that regulatory guidance has avoided prescribing of any one AMA framework, including the LDA, even though the LDA has become the de facto choice among AMA institutions, including those that have recently exited parallel run.

- It must be easily understood and implemented using any widely available statistical software package.
- It must implementable using only a reasonably powerful desktop or laptop computer.
- It must provide unambiguous improvements over the most widely used implementations of LDA on all three of the key criteria for assessing the efficacy of an operational risk capital estimation framework: capital accuracy, capital precision, and capital robustness

The remainder of the paper is organized as follows. First, I define and discuss Jensen's inequality and its apparent effects on operational risk capital estimation under LDA, demonstrate the conditions under which these effects are material, and define the extremely wide range of (severity parameter) estimators for which these results are relevant. Next I develop and present the Reduced-bias Capital Estimator (RCE), discuss the details of its implementation, and present some new analytic derivations that assist in this implementation (as well as with the implementation of LDA generally). Thirdly, I conduct an extensive simulation study comparing RCE to the most widely used implementation of LDA as a benchmark (i.e. using maximum likelihood estimation (MLE)).[20] The study covers a number of very distinct severity distributions, both truncated and non-truncated, widely varying values for regulatory capital (RCap) and economic capital (ECap) at the unit-of-measure level (from $38m to over $10.6b), and wide ranges of severity parameter values that cover conditions of both finite and infinite severity mean (showing that RCE "works" even under the latter condition). The study also includes i) a new analytic derivation for the mean of a very commonly used severity distribution under truncation; ii) a very fast, efficient, and stable sampling (perturbation) method based on iso-densities; iii) an improved single loss approximation for estimating capital under conditions that may include infinite means; and iv) a new analytic approximation of the Fisher information a commonly used severity distribution under truncation (thus avoiding computationally expensive numeric integration). I discuss throughout how RCE is entirely consistent with the LDA framework specifically, and with regulatory intent and expectation generally regarding its responsible (i.e. unbiased, or close) implementation. I conclude with a summary and a discussion of areas for future research.

Key Methodological Background

Before discussing Jensen's inequality, I turn to a more recent result to provide some explanatory foundation for the relevance of the former in operational risk capital estimation. As mentioned above, under LDA the aggregate loss distribution is defined as the convolution of the frequency and severity distributions, and in

[20] For severity distribution estimation, AMAG (2013), in its range of practice survey from 2012, states "MLE is predominant, by far." This also is true for other components of the framework (e.g. dependence modeling across UoMs).

It is important to note that the MLE-based capital distributions do not dramatically differ from those of most other (severity) estimators in this setting, and so the sometimes enormous benefits of RCE over MLE also apply to most other implementations of LDA.

almost all cases no closed-form solutions exist to estimate the VaR of this compound distribution. Böcker and Klüppelberg (2005) and Böcker and Sprittulla (2006) were the first to provide an analytical approximation of this VaR in (1), and Degen (2010) refined this and expanded its application to include conditions of infinite mean in (2a,b,c).[21]

$$C_\alpha \approx F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)+(\lambda-1)\mu \tag{1}$$

$$\text{if } \xi<1, \qquad C_\alpha \approx F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)+\lambda\mu \tag{2.a}$$

$$\text{if } \xi=1, \qquad C_\alpha \approx F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)+c_\xi\lambda\mu_F\left[F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)\right] \tag{2.b}$$

$$\text{if } 1<\xi<2, \quad C_\alpha \approx F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)-(1-\alpha)F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right)\cdot\left(\frac{c_\xi}{1-1/\xi}\right) \tag{2.c}$$

($\xi \geq 2$ is so extreme as to not be relevant here) where, $c_\xi = (1-\xi)\dfrac{\Gamma^2(1-1/\xi)}{2\Gamma(1-2/\xi)}$ for $1<\xi<\infty$ and

$c_\xi = 1$ for $\xi = 1$, $\mu_F(x) = \int_0^x \left[1-F(s)\right]ds$; C_α = "capital"; $\alpha =$ "confidence level" (e.g. $\alpha = 0.999$ for RCap);

$F^{-1}(\ )$ is the quantile function of the severity; λ is the (typically Poisson) frequency parameter; μ= mean of severity; ξ= the tail index; and $\Gamma(\ )$ is the gamma function.

I focus now on Degen's (2.a) to make the point that the first term, the severity quantile, is much larger – sometimes even orders of magnitude larger – than the second term (the "mean correction"), and so capital is essentially a very large quantile of the severity distribution (and this is consistent with the widely cited finding in the literature that severity, not frequency, is what really drives capital (see Opdyke and Cavallo, 2012a and 2012b)). But at least as important is the fact that the quantile of the severity distribution that must be estimated is much larger than that corresponding to the 99.9%tile – it actually corresponds to the [1 – (1-α)/λ] = 0.99997 = 99.997%tile (assuming λ=30), which is nearly two orders of magnitude larger (the corresponding percentiles for ECap are the 99.97%tile and 99.999%tile, respectively, assuming λ=30 and a good credit rating). So not only is capital essentially a quantile of the severity distribution, but this quantile also is extremely large. The essence of the problem, then, reduces to estimating an extremely large quantile of the severity distribution.[22] This fact, *combined* with the fact that the only severities used (and allowed) in operational risk capital estimation are

---

[21] Sahay et al. (2007) presented similar results a few years earlier.

[22] However, as noted above, estimation and simulation of the frequency parameter is never ignored in this paper. The purpose of making this point here is heuristic as it pertains to the explanation of the relevance of Jensen's inequality in this setting.

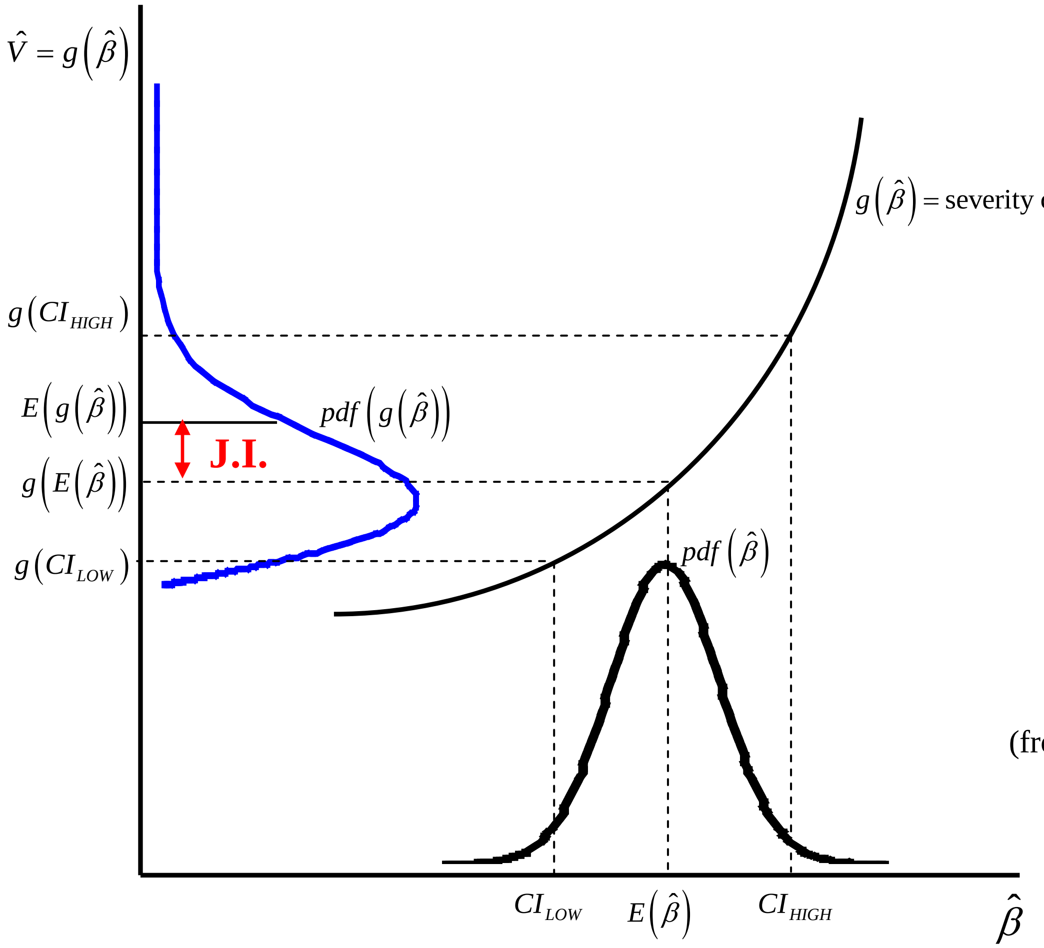
medium- to heavy-tailed, is the reason that Jensen's inequality apparently can so adversely and materially affect capital estimation, as described below.

Jensen's Inequality

Jensen's Inequality Defined

In 1906, Johan Jensen proved that the (strictly) convex transformation of a mean is less than the mean after a (strictly) convex transformation (and that the opposite is true for strictly concave transformations). When applied to random variable β, this is shown in Figure 1 below as E[g($\hat{\beta}$)] > g(E[$\hat{\beta}$]), with a magnitude of

FIGURE 1: Graph of Jensen's Inequality with Strictly Convex Function (right-skewed, heavy-tailed cdf)



(from Kennedy, 1992. p.37)

Jensen's Inequality = J.I. = E[g($\hat{\beta}$)] – g(E[$\hat{\beta}$]).[23] An intuitive interpretation of Figure 1 is that the strictly convex function, g( ), "stretches out" the values of the random variable β above its median more than it does below, thus positively skewing the distribution of $\hat{V}$ = g($\hat{\beta}$) and increasing its mean above what it would have

[23] Figure 1 shows VaR for a given cumulative probability, *p*. As *p* increases beyond some large level (e.g. *p* > 0.999), so does VaR's convexity in this setting, as discussed later in the paper.

been had the function g( ) been a linear function. In other words, $\hat{V}$ also would have been symmetric like $\hat{\beta}$, with a mean equal to its median, but because g( ) is convex, its upper tail is "stretched out" making its mean greater than its median.[24]

Jensen's Inequality in Operational Risk Capital Estimation

The relevance of Jensen's inequality to operational risk capital estimation appears to be the joint fact that the only severities used (and permitted) in operational risk capital estimation are medium- to heavy-tailed, *and* the severity quantile being estimated is so extremely large: under these conditions, VaR appears to always be a convex function, like g( ), of the parameters of the severity distribution, which here is the vector β (we can visualize β as a single parameter without loss of generality as the multivariate case for Jensen's inequality is well established (see Schaefer, 1976). Consequently, the capital estimation, $\hat{V}$ = g($\hat{\beta}$), will be biased upwards. In other words, its expected value, E[g($\hat{\beta}$)], will be larger than its true value, g(E[$\hat{\beta}$]). Stated differently, if we generated 1,000 i.i.d. random samples of losses based on "true" severity parameter values = β, and for each of the 1,000 estimated $\hat{\beta}$'s we calculated capital $\hat{V}$ = g($\hat{\beta}$), the average of these 1,000 capital estimates ($\hat{V}$'s) will be larger than V =g(β), which is "true" capital.

The above is straightforward, and the biasing effects of Jensen's inequality are very well established and not in doubt. The only question is whether VaR *always* is a strictly convex function of the estimators of the severity parameters. All of the estimators used in this setting are at least symmetrically distributed, and most are normally distributed, at least asymptotically.[25] So if VaR is a convex function of them, there is no doubt that capital will be systematically biased upwards (in addition to being, on average, more skewed, and with larger root mean squared error (RMSE)[26] and standard deviation, as shown empirically later in this paper). To check for this convexity, we can do several things: examine VaR as a function of each parameter individually (i.e.

---

[24] Importantly, note that the median of $\hat{V}$ actually is equal to the transformation of the original mean: g(E[$\hat{\beta}$]) = g(β) . This is due to the fact that g( ) is a monotone transformation (here, of a symmetric, unbiased variable). This is shown below and exploited to our advantage later in this paper when designing a reduced-based capital estimator.

[25] All M-class estimators are asymptotically normal, and these include many of the most commonly used estimators in this setting (e.g. maximum likelihood estimation (MLE), many generalized method of moments (GMM) estimators, penalized maximum likelihood (PML), optimally bias-robust estimator (OBRE), Cramér von-Mises (CvM) estimator, and PITS estimator, among many others). See Hampel et al. (1986) and Huber and Ronchetti (2009) for more details.

[26] The MSE is the average of the squared deviations of a random variable from its true value. This is also equal to the variance of the random variable plus its bias squared $MSE = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{V}_i - V\right)^2 = Var\left(\hat{V}\right) + \left[Bias\left(\hat{V}\right)\right]^2$. The RMSE is the square root of MSE. So RMSE of the capital distribution = $RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\hat{V}_i - V\right)^2}$

check for marginal convexity); examine and attempt to define the multidimensional surface of VaR as a multivariate function of the severity parameters (i.e. check for multidimensional convexity (in three dimensional space for two-parameter severities)); and examine the behavior of VaR itself under straightforward i.i.d. Monte Carlo simulations to determine if it is consistent with the effects of Jensen's inequality as a convex, or at least "convex-dominant" function of the severity parameters.[27]

The check for marginal convexity has been performed graphically in Appendix A Figure A1, for three widely used severity distributions (7 others – the three-parameter Burr Type XII, the LogLogistic, and the truncated versions of all five – are available from the author upon request).[28] All demonstrate that for *sufficiently extreme* percentiles (e.g. $p > 0.999$), VaR is a convex function of either one or both of the severity parameters (and a linear function of the others). These results are summarized in Table 1.

One approach to checking for convexity (or convexity-dominance) in the multidimensional VaR surface is an examination of the signs and relative magnitudes of the eigenvalues of the shape operator (see Jiao and Zha, 2008). This turns out to be analytically nontrivial, if not intractable under truncation, and even numeric calculations for many of the relevant severities are nontrivial given the sizes of the severity percentiles (e.g. $p = 0.99999$) that must be used in this setting (because most of the gradients are exceedingly large for such high percentiles). So this research currently remains underway, and without this mathematical verification, attributions of capital inflation *apparently* due to Jensen's inequality and VaR's *apparent* convexity are based solely on empirical results, and conservatively and explicitly deemed "preliminary" or "presumed" herein.

However, arguably the most directly relevant of these three "checks" is the behavior of the capital estimate itself: if it consistently reflects what we would expect to see under Jensen's inequality, i.e. systematically inflated capital estimates under i.i.d. Monte Carlo simulations, then this, combined with consistent marginal convexity, would provide reasonably strong, if preliminary evidence that VaR is a convex (or convex-dominant) function of the entire vector of severity parameters. As such, severity parameter *estimates* that are subject to sampling variability will generate capital estimates that are, on average, inflated, as shown in Figure 1. And this

---

[27] "Convex-dominant" is used here to indicate cases where VaR is not a convex function of each parameter individually, but may be a convex function of the entire vector of severity parameter estimates, given its variance-covariance matrix. For example, while VaR of GPD is marginally convex in ξ, it is marginally linear in θ (see Appendix A). Also, some areas of its multidimensional surface appear to indicate the existence of saddlepoints, i.e. surfaces with hyperbolic points. But if the convexity in one direction of such a surface is orders of magnitude larger than the concavity in the other, as measured by the relative sizes of their principal curvatures (i.e. the eigenvalues of its shape operator), then the net effect of sampling variation on VaR, under the estimator's variance-covariance matrix, would appear to be dominated by convexity rather than concavity.

[28] This is easily confirmed analytically for those distributions that have closed form representations of their inverse CDFs (i.e. VaR functions). For example, for the LogNormal, $VaR = \exp\left(\mu + \sigma\Phi^{-1}\left(p\right)\right)$; $\partial^2 VaR/\partial\mu^2 = VaR$; $\partial^2 VaR/\partial\sigma^2 = VaR\cdot\left[\Phi^{-1}\left(p\right)\right]^2$.

is exactly what we observe: consistent marginal convexity, as shown in Appendix A, and consistent and strong capital inflation, as shown in the extensive simulation study presented below. But the broader question here is whether *all* severity distributions relevant to operational risk capital estimation can be so characterized.

TABLE 1: Marginal VaR Behavior OVER RELEVANT DOMAIN (p > 0.999) by Severity

Severity Distribution	**VaR is a Convex/Linear Function of…**			**Relationship Between Parameters**
	Parameter 1	Parameter 2	Parameter 3	
1) LogNormal (μ, σ)	Convex	Convex		Independent
2) LogLogistic (α, β)	Linear	Convex		Independent
3) LogGamma (a, b)	Convex	Convex		Dependent
4) GPD (ξ, θ)	Convex	Linear		Dependent
5) Burr (type XII) (Υ, α, β)	Convex	Convex	Linear	Dependent
6) Truncated 1)	Convex	Convex		Dependent
7) Truncated 2)	Linear	Convex		Dependent
8) Truncated 3)	Convex	Convex		Dependent
9) Truncated 4)	Convex	Linear		Dependent
10) Truncated 5)	Convex	Convex	Linear	Dependent

Before answering this question, it should be noted here that convexity sometimes replaces subadditivity (as well as positive homogeneity; see Fölmer and Schied, 2002, and Frittelli and Gianin, 2002) as an axiom of coherent risk measures (see Artzner et al., 1999), and is only slightly less strong an axiom compared to subadditivity.[29] And while it is very well established that VaR is not *globally* subadditive across all quantiles for all parametric statistical distributions, for the specific group of medium- to heavy-tailed severities relevant to LDA-based operational risk capital estimation, *and* very extreme percentiles of those severities ($p > 0.999$), it appears that VaR may very well always be subadditive. Danielsson et al. (2005) proved that regularly-varying severities with finite means all were subadditive for sufficiently high percentiles (e.g. $p > 0.99$; for similar results, see also Embrechts and Neslehova, 2006, Ibragimov, 2008, and Hyung and de Vries, 2007). And the same result has been shown empirically in a number of publications (see, for example, Degen et al., 2007). Although supra-additivity has been proven for some families of extremely heavy-tailed severities with infinite mean, (see Embrechts and Nešlehová, 2006, Ibragimov, 2008, and Hyung and de Vries, 2007), and consequently strong caution has been urged when using such models for operational risk capital estimation (see Nešlehová et al., 2006), this does not cover all such severities. In fact, high VaR ($p > 0.999$) of the Generalized Pareto Distribution (GPD) with infinite mean ($\theta = 40{,}000$ and $\xi = 1.1$) is shown in Appendix B, Figure B1 to still be a

[29] In fact, for any normalized risk measure, the presence of any two of the three properties of convexity, subadditivity, and positive homogeneity implies remaining third (see Föllmer and Schied, 2011).

convex function of ξ and a linear function of θ. And corresponding capital simulations in Appendix B (Table B1) demonstrate continued and notable capital bias consistent with Jensen's inequality, infinite mean notwithstanding (capital bias of more than 80% and more than 120% over true capital for RCap and ECap, respectively). These easily replicated results demonstrate that supra-additivity is not a given for very heavy-tailed severities with some infinite moments, at least for certain parameter values. What's more, many practitioners in this setting restrict severities, or severity parameter values, to those indicating finite mean, arguing that allowing expected losses to be infinite makes no sense for an operational risk capital framework. This would make moot the issue of the possible supra-additivity of the severity. Others counter that regulatory requirements dictate the estimation of quantiles, not moments, and that capital models, from a robustness perspective, should remain agnostic regarding the specific characteristics of a loss distribution's moments.

Regardless of the position one takes on this debate, a mathematical proof of VaR's subadditivity or convexity for all severities relevant to operational risk capital estimation (a group that is not strictly defined) is beyond the scope of this paper. However, while undoubtedly useful, this is not strictly necessary here, because the number of such severities in this setting is finite, and checking the subset of those in use by any given financial institution, one by one, is very simple to do graphically, as was done in Appendix A, Figure A1. Graphical checks can be complemented with a simple simulation study wherein capital is estimated, say, 1,000 times based on i.i.d. samples generated from a chosen severity. If the mean of these 1,000 capital estimates is noticeably larger than the "true" capital based on the "true" severity parameters (where the original parameter estimates are treated as "true"), and this is consistent with graphing VaR as a function of the parameter values, then attributing this systemic bias to Jensen's inequality remains the most plausible, if not highly probable explanation.[30]

Note that it is just as easy to demonstrate the opposite, too, for a given severity. For example, VaR of the Gaussian (Normal) distribution is a linear function of both of the distribution's parameters, μ and σ. These marginal results indicate that Jensen's inequality likely could never affect capital estimation based on this distribution. This is shown both graphically in Appendix B, Figure B1, as well as via capital simulation in Table B1 in Appendix B,[31] which shows no positive capital bias,[32] even for the extremely large quantiles that are estimated under LDA. Remember, however, that the normal distribution, whether truncated or not, is far too

[30] This assumes, of course, that any approximations used to estimate capital are correct and reasonably accurate, and that the simulated data is i.i.d. to remove any other potential source of bias. See discussion of the former point below.

[31] This simulation ignores the need for truncation of the normal distribution at zero as the findings do not change.

[32] Very slight negative capital bias due to the estimation of λ, the frequency parameter, is discussed below.

light-tailed to be considered for use in operational risk capital estimation. And this demonstrates that *both* characteristics – the medium- to heavy-tailed nature of the severity, *and* estimation of its very high percentiles (e.g. $p > 0.999$) – are required simultaneously for the presumed convexity of VaR to manifest, and thus, for Jensen's inequality to bias capital estimates.

To conclude this section on the biasing effects on LDA-based capital most likely attributable to Jensen's inequality, we must address the effects of λ on capital, both in the first terms of (2a,b,c) as well as the subsequent "correction" terms. Recall that λ is the parameter of the frequency distribution, whose default is the Poisson distribution.[33] For the extremely wide range of severity and frequency parameter values examined in this paper, capital actually is a concave function of λ, but its (negative) biasing effects on capital estimation are very small, if not de minimis. This is shown in 216 simulation studies summarized in Table C1 in Appendix C wherein λ is the only stochastic component of the capital estimate.[34] Bias due only to λ always is negative, but rarely exceeds -1%, and then just barely. So for all practical purposes VaR is essentially a linear function of λ in this setting, and any (negative) biasing effect on capital is swamped by the much larger (positive) biasing effect of the severity parameters on capital, as shown in the Results section below. And regardless, RCE takes the net effect of both sources of bias into account, as discussed below.

When Are the Presumed Effects of Jensen's Inequality Material?

When VaR is a convex function of the vector of severity parameters, capital estimates will be biased upwards – always. But when is this capital inflation material? The most straightforward and reasonable metric for materiality is the size of the bias, both relative to true capital and in absolute terms. A bias of, say, \$0.5m when true capital is \$250m arguably is not worth the concern of those estimating capital (especially if its standard deviation is, say, \$400m, which is actually somewhat conservative). However, it would be hard to argue that a bias of \$200m, \$75m, or even \$25m was not worth the trouble to address statistically and attempt to at least mitigate it, if not eliminate it. And in addition to bias that sometimes exceeds 100% of true capital, the dramatic increase in the skewness and spread of the distribution of capital estimates (as shown in the simulation study below) alone could be reason enough to justify the development and use of a statistical method to eliminate it, especially if its implementation is relatively straightforward and fast.

[33] Empirically there is rarely much difference in capital regardless of the frequency distribution chosen, and the Poisson is mathematically convenient as well, so it had become the widely used default. Also note that (2.a,b,c) require only slight modification to accommodate other reasonable, non-Poisson frequency distributions, such as the Negative Binomial.

[34] These simulations cover all severity conditions, and most sample sizes, under which LDA-MLE and RCE are tested later in the paper.

It turns out there are three factors that contribute to the size of the capital bias (and the other abovementioned effects on the capital distribution): a) the size of the variance of the severity parameter estimator; b) the heaviness of the tail of the given severity distribution; and c) the size of the quantile being estimated. Directionally, larger estimator variance is associated with larger bias; heavier tails are associated with larger bias, and more extreme quantiles are associated with larger bias. Typically a) is driven most by sample size, and because larger sample sizes are almost always associated with smaller estimator variance, larger samples are associated with smaller bias. The choice of severity, typically determined by goodness-of-fit tests,[35] along with the size of its estimated parameter values drive b). So for example, truncated distributions, all else equal, will exhibit more bias than their non-truncated counterparts (with the same parameter values). And the choice of quantile, c), is determined by α in formula (2.a), and α is set at 0.999 for regulatory capital (and typically α = 0.9997, or close, for economic capital, depending on the institution's credit rating). So ECap will exhibit larger capital bias than RCap, all else equal.

All of these factors, and the directions of their effects, are consistent with the effects of Jensen's inequality, and with VaR as a convex (or convex-dominant) function of the severity parameter estimates, All three, but particularly a), can be visualized with Figure 1. The smaller the variance of the estimator of the severity parameter, β, on the X-axis, the less the values of $g\left(\hat{\beta}\right)$ can be stretched out above the median, all else equal, and so the less capital estimates will exhibit bias. In the extreme, if there is no variance, then all we have is β, the true severity parameter, and there is no bias in our capital estimate (because it is no longer an estimate – it is true capital). For b) and c), heavier tails, and more extreme quantiles of those tails, both are associated with greater convexity as shown in Appendix A, Figure A1, so g( ) will be more curved and will "stretch out" the capital estimates more and increase bias, all else equal.

The effects of sample size on capital bias are shown empirically in Table 2 for sample sizes of approximately 150, 250, 500, 750, and 1,000,[36] corresponding to λ = 15, 25, 50, 75, and 100, respectively, for a ten year period. The size of the bias relative to true capital is (almost) always greater when the number of operational

[35] In this setting these tests typically are empirical distribution function-based (EDF-based) statistics, based on the difference between the estimated cumulative distribution function (CDF) and the EDF. The most commonly used here are the Kolmogorov-Smirnov (KS), the Anderson-Darling (AD), and the Cramér-von Mises (CvM) tests.

[36] These are approximate sample sizes because the annual frequency, of course, is a random variable (i.e. λ is stochastic). Because the Poisson distribution is used for this purpose, the standard deviation of the number of losses is, annually, StdDev = $\sqrt{\lambda}$, and for a given number of years, StdDev = $\sqrt{\#\,years \cdot \lambda}$.

risk loss events in the sample is smaller.[37] Unfortunately, UoM's with thousands of loss events are not nearly as common as those with a couple of hundred loss events, or less. So from an empirical perspective, we are squarely in the bias-zone: bias is material for many, if not most estimations of capital at the UoM level.[38] In fact, this is exactly what Ergashev et al. (2014) found in their study comparing capital based on shifted vs. truncated lognormal severity distributions. The latter exhibited notable bias that disappeared as sample sizes increased up to n = 1,000, exactly as in the simulation study in this paper. However, the authors did not attribute this empirical effect to an analytical result (i.e. Jensen's inequality), as is done here.

It is important to explicitly note here the converse, that is, the conditions under which LDA-MLE-based capital bias apparently due to Jensen's inequality is *not* material. This is shown empirically in Table 2 and in the simulation study below, but general guidelines include a) *sample sizes*: sample sizes in the low hundreds, which are most common for operational risk loss event data, will exhibit notable bias, all else equal, while those in the thousands typically will exhibit much more modest, if any bias, depending on the severity (see Table 2 – three severities exhibit very little bias for $n \approx 1{,}000$ ($\lambda = 100$), while two others exhibit noticeable but arguably modest bias of around 20% over true capital, and the last exhibits 5%-20% bias, depending on the parameter values). b) *severities*: certain severities are more heavy-tailed than others (e.g. LogGamma is more heavy-tailed than LogNormal, and GPD is more heavy-tailed than LogGamma), and truncated severities, by definition, are heavier-tailed distributions than their non-truncated counterparts, all else equal (that is, with the same parameter values). c) *parameter values*: note that VaR sometimes is a convex function of only one of the parameters of the distribution (for example, as shown in Appendix A, Figure A1, for the GPD and Truncated GPD distributions VaR is linear in θ but convex in ξ), so the magnitude of capital bias, apparently, will hinge primarily on the magnitude of this parameter, all else equal. This can be seen for almost all cases of the GPD and Truncated GPD distributions in Table 2. Capital is approximately equal in the paired, adjacent rows for these severities, yet bias is larger for the second row of the pair, where ξ is always larger. The only exception is where $\lambda = 15$ for the Truncated GPD, because sometimes the smaller number of losses decreases capital, on average, via the decrease in the quantile of the first term of (2.a) more than it increases capital, on average, due to an increase in parameter variance, so that on net, capital bias actually decreases even though ξ is slightly larger.

---

[37] The one exception is the one case (LogNormal, $\mu = 10$, $\sigma = 2$) where the smaller sample size ($n \approx 150$) decreases capital, on average, via the decrease in the percentile of the first term of (2.a) more than it increases capital, on average, due to an increase in parameter variance, so that on net, capital bias actually decreases very slightly.

[38] Again, this is also confirmed in RMA (2013), which cites the need for "Techniques to remove or mitigate the systematic overstatement (bias) of capital arising in the context of capital estimation with the LDA methodology"

TABLE 2: MLE Capital Bias Beyond True Capital by Sample Size by Severity by Parameter Values

Severity Dist.	Parm1	Parm2	RCap % Bias λ = 15	RCap % Bias λ = 25	RCap % Bias λ = 50	RCap % Bias λ = 75	RCap % Bias λ = 100	ECap % Bias λ = 15	ECap % Bias λ = 25	ECap % Bias λ = 50	ECap % Bias λ = 75	ECap % Bias λ = 100
	μ	**σ**										
LogN	10	2	6.0%	6.7%	3.0%	1.5%	1.5%	7.3%	7.8%	3.5%	1.8%	1.8%
LogN	7.7	2.55	11.9%	11.5%	5.4%	3.0%	2.8%	14.2%	13.2%	6.2%	3.4%	3.3%
LogN	10.4	2.5	11.3%	11.0%	5.1%	2.8%	2.7%	13.5%	12.7%	5.9%	3.2%	3.1%
LogN	9.27	2.77	14.9%	13.8%	6.5%	3.7%	3.4%	17.6%	15.8%	7.5%	4.2%	3.9%
LogN	10.75	2.7	13.9%	13.1%	6.2%	3.4%	3.2%	16.5%	15.0%	7.1%	3.9%	3.7%
LogN	9.63	2.97	17.9%	16.1%	7.7%	4.4%	4.0%	21.1%	18.5%	8.8%	5.0%	4.6%
TLogN	10.2	1.95	18.9%	11.5%	8.1%	3.6%	2.9%	24.6%	14.7%	10.1%	4.6%	3.7%
TLogN	9	2.2	52.0%	26.5%	13.9%	7.3%	5.3%	76.8%	35.0%	17.7%	9.5%	6.9%
TLogN	10.7	2.385	42.9%	26.4%	12.5%	6.0%	5.2%	57.2%	32.4%	15.2%	7.4%	6.4%
TLogN	9.4	2.65	64.2%	39.1%	20.0%	13.9%	8.4%	87.8%	51.6%	24.8%	17.0%	10.3%
TLogN	11	2.6	49.9%	27.1%	14.8%	9.2%	5.6%	63.6%	34.0%	17.7%	11.0%	6.8%
TLogN	10	2.8	90.9%	40.2%	17.1%	13.2%	8.8%	127.3%	51.5%	21.1%	16.1%	10.8%
	a	**b**										
Logg	24	2.65	22.3%	13.6%	5.6%	4.4%	1.1%	28.3%	17.0%	7.0%	5.4%	1.7%
Logg	33	3.3	17.8%	8.5%	3.6%	3.2%	0.4%	22.2%	10.7%	4.6%	4.0%	0.7%
Logg	25	2.5	26.4%	15.7%	8.3%	5.8%	1.3%	33.3%	19.5%	10.1%	7.0%	1.9%
Logg	34.5	3.15	16.3%	10.9%	6.3%	4.0%	0.6%	20.5%	13.5%	7.7%	4.8%	1.0%
Logg	25.25	2.45	27.9%	18.3%	9.5%	5.2%	1.6%	35.2%	22.5%	11.6%	6.4%	2.2%
Logg	34.7	3.07	19.3%	13.7%	7.1%	3.3%	0.4%	24.2%	16.8%	8.6%	4.1%	0.8%
TLogg	23.5	2.65	166.7%	56.1%	31.7%	14.6%	13.5%	329.3%	83.1%	45.0%	20.1%	18.5%
TLogg	33	3.3	72.7%	34.1%	13.2%	7.7%	6.6%	110.5%	46.1%	17.7%	10.3%	8.8%
TLogg	24.5	2.5	110.2%	60.4%	25.8%	16.9%	9.9%	169.5%	84.9%	34.2%	22.4%	13.3%
TLogg	34.5	3.15	45.3%	24.5%	11.6%	7.7%	4.8%	63.3%	32.2%	15.0%	9.8%	6.3%
TLogg	24.75	2.45	102.1%	62.9%	23.4%	16.0%	9.9%	152.3%	87.6%	31.2%	20.6%	13.2%
TLogg	34.6	3.07	40.7%	24.3%	13.6%	8.3%	4.3%	55.0%	31.8%	17.0%	10.3%	5.7%
	ξ	**ϑ**										
GPD	0.8	35000	80.3%	56.9%	30.5%	17.6%	14.0%	119.9%	81.9%	41.5%	23.3%	18.6%
GPD	0.95	7500	108.8%	75.6%	39.8%	23.0%	18.2%	163.4%	109.2%	54.0%	30.2%	23.9%
GPD	0.875	47500	91.1%	63.7%	34.8%	20.0%	16.1%	135.9%	91.9%	47.3%	26.5%	21.3%
GPD	0.95	25000	105.7%	73.2%	39.7%	22.8%	18.3%	158.8%	105.9%	53.8%	30.0%	24.1%
GPD	0.925	50000	90.0%	67.8%	37.4%	21.8%	17.3%	137.6%	97.9%	50.8%	28.7%	22.8%
GPD	0.99	27500	109.5%	76.4%	41.6%	24.3%	19.3%	164.9%	110.7%	56.5%	31.9%	25.3%
TGPD	0.775	33500	81.6%	52.0%	25.3%	17.7%	14.4%	127.8%	75.7%	34.7%	23.9%	19.1%
TGPD	0.8	25000	71.3%	56.9%	28.3%	19.6%	16.0%	108.5%	82.9%	38.8%	26.5%	20.9%
TGPD	0.868	50000	101.2%	63.0%	33.1%	20.6%	15.8%	154.8%	92.0%	45.5%	27.7%	20.7%
TGPD	0.91	31000	93.8%	68.6%	34.1%	23.2%	17.8%	146.7%	100.4%	46.3%	30.9%	23.2%
TGPD	0.92	47500	115.9%	64.7%	35.7%	24.0%	17.1%	176.7%	93.9%	48.6%	32.0%	22.5%
TGPD	0.95	35000	105.6%	68.2%	39.0%	24.6%	19.1%	168.6%	100.8%	53.7%	32.8%	25.1%

*NOTE: #simulations = 1,000; time period = 10 years; for RCap and ECap, **α** = 0.999 and 0.9997, respectively.

Unfortunately there currently are no formulaic rules to determine whether LDA-MLE-based capital bias is material for a given sample of loss event data (and the best-fitting severity chosen), because all of these three factors – a), b), and c) – interact in ways that are not straightforward. And materiality is a subjective assessment as well. So the only way to answer this question of materiality is to conduct a simple simulation given the estimated values of the severity (and frequency) parameters: i) treat the estimated parameter values as "true" and calculate "true" capital; ii) use the "true" parameter values to simulate 1,000 i.i.d. data samples and for each of these samples, re-estimate the parameter values and calculate capital for each sample; iii) compare the mean of these 1,000 capital estimates to "true" capital, and if the (positive) difference is large or at least notable, then capital bias is material.[39] This is, in fact, exactly what was done for Table 2, which is taken from the simulation study presented later in this paper.

Estimators Apparently Affected by Jensen's Inequality

There are a wide range of estimators that have been brought to bear on the problem of estimating severity distribution parameters in this setting. Examples include maximum likelihood estimation (MLE; see Opdyke and Cavallo, 2012a and 2012b), penalized likelihood estimation (PLE; see Cope, 2011), Method of Moments (see Dutta and Perry, 2007) and Generalized Method of Moments (see RMA, 2013), Probability Weighted Moments (PWM – see BCBS, 2011), Bayesian estimators (with informative, non-informative, and flat priors; see Zhou et al., 2013), extreme value theory – peaks over threshold estimator (EVT-POT; see Chavez-Demoulez et al., 2013),[40] robust estimators such as the Quantile Distance estimator (QD; see Ergashev, 2008), Optimal Bias-Robust Estimator (OBRE; see Opdyke and Cavallo, 2012a), Cramér-von Mises estimator (CvM – not to be confused with the goodness-of-fit test by the same name; see Opdyke and Cavallo, 2012a), Generalized Median Estimator (see Serfling, 2002, and Wilde and Grimshaw, 2013), PITS Estimator (only for Pareto severity; see Finkelstein et al., 2006), and many others of the wide class of M-Class estimators (see Hampel et al., 1986, and Huber and Ronchetti, 2009). Which of these generate inflated capital estimates, apparently due to Jensen's inequality? The answer is any that would be represented as β on Figure 1, which is

[39] It is possible, of course, that the original estimated parameter values based on actual loss data are much larger than the "true" but unobservable parameter values due simply to random sampling error, in which case bias may not be material. But even in this case, the parameter values actually used to estimate capital will be the (high) estimates, because these are the best we have: we will never know the "true" values because we have only samples of loss data, not a population of loss data. And so bias will be material based on these estimated parameter values and the given sample of loss event data. Over time, unbiased estimates based on larger samples of data will converge (asymptotically) to true parameter values.

[40] Although estimating operational risk capital via EVT-POT was not explicitly tested in this paper for capital bias induced by VaR's apparent convexity, it would appear to be subject to the same effects. This approach relies on extreme value theory to estimate only the tail of the loss distribution which, beyond some high threshold, asymptotically converges to a GPD distribution (see Rocco, 2011, and Andreev et al., 2009). The estimated parameters of the GPD distribution, however, are generally unbiased (especially if specifically designed unbiased estimators are used in the case of very small samples; for example, see Pontines and Siregar, 2008). As such, they can be represented on Figure 1 as β, and thus would provide biased VaR estimates. See Chavez-Demoulez et al. (2013) for a rigorous application of EVT-POT to operational risk capital estimation.

to say, apparently all of them.[41] All the relevant estimators at least will be symmetrically distributed, and many, if not most, will be normally distributed, at least asymptotically (like all M-Class estimators). But normality most certainly is not a requirement for this bias to manifest (and even symmetry is not a requirement), and so capital based on all of these estimators will be subject to the same biasing effects outlined above. There is some evidence that robust estimators generate capital estimates that are less biased than their non-robust counterparts, and while this intuitively makes sense, unfortunately the mitigating effect on capital bias does not appear to be large (for some empirical results, see Opdyke and Cavallo, 2012a; Opdyke, 2013; and Joris, 2013). To the extent that there are differences in the size of the capital bias associated with each of these estimators, the size of the variance will be the main driver, but given the (maximal) efficiency of MLE,[42] it is safe to say that none of these other estimators will fare much better, if at all, regarding LDA-based capital bias, ceteris paribus.

Severities Apparently Affected by Jensen's Inequality

As discussed above, it appears that all severities commonly used in operational risk capital estimation satisfy the criteria of being heavy-tailed enough, and simultaneously the quantile being estimated is extreme enough, that the capital estimates they generate are upwardly biased. A number of papers have proposed using mixtures of severities in this setting, but as shown in Joris (2013), capital estimates based on these, too, appear to exhibit notable bias. Another common variant is to use spliced severities, wherein one distribution is used for the body of the losses and another is used for the right tail (see Ergashev, 2009, and RMA, 2013), and often the splice point is endogenized. Sometimes the empirical distribution is used for the body of the severity, and a parametric distribution is used for the tail. For these cases, too, we can say that as long as the ultimate estimates of the tail parameter can be represented as β in Figure 1, the corresponding capital estimates also will exhibit the same systematic inflation. A simulation study testing the latter of these cases is beyond the scope of the current paper, but would be very useful to confirm results for spliced distributions similar to those of Joris (2013) for mixed distributions.

Reduced-bias Capital Estimator

Note that as mentioned above, the median of the capital distribution, *if sampled from a distribution centered on the true parameter values*, is an unbiased estimator of true capital, as shown below:

[41] One distinct approach proposed for operational risk capital estimation that may diverge from this paradigm is the semi-parametric kernel transformation (see Gustafsson and Nielson, 2008, Bolancé et al., 2012, and Bolancé et al., 2013). However, in a closely related paper, Alemany, Bolancé and Guillén (2012) discuss how variance reduction in their double transformation kernel estimation of VaR increases bias. In contrast, RCE simultaneously decreases both variance and bias in the VaR (capital) estimate.

[42] Of course, MLE achieves the maximally efficient Cramer-Rao lower bound only under i.i.d. data sample conditions.

From Figure 1, if $\hat{\beta}$ is symmetrically distributed and centered on true β (that is, $\hat{\beta}$ is unbiased, as is the case, asymptotically, for MLE under i.i.d. data), then:

$E\left(\hat{\beta}\right) = F^{-1}\left(0.5\right)$, i.e. the mean equals the median, so

$$g\left[E\left(\hat{\beta}\right)\right] = g\left[F^{-1}\left(0.5\right)\right]$$

And as g[ ] is strictly convex and a monotonic transformation,

$$g\left[E\left(\hat{\beta}\right)\right] = g\left[F^{-1}\left(0.5\right)\right] = G^{-1}\left(0.5\right).$$

So as long as $\hat{\beta}$ is unbiased, the median of the capital distribution is an unbiased estimator of capital. In other words, given a monotonic and strictly convex transformation function (i.e. g( ), or VaR), the median of the transformed variable (i.e. capital estimates) is equal to the transformation of the original mean (i.e. $G^{-1}\left(0.5\right) = g\left(E\left[\hat{\beta}\right]\right) = g\left(\beta\right)$]) of a symmetric, unbiased variable (e.g. MLE estimates of severity parameters under i.i.d. data). However, this begs the question of unbiased capital estimation, because in reality we have only one sample and one corresponding vector of (estimated) parameter values, $\hat{\beta}$, and these will never exactly equal the true severity parameter values, β, of the underlying data generating process. So simply taking the median of the capital distribution will not work. But this relationship still can be exploited in constructing a reduced-bias capital estimator, as shown below.

The motivation behind the development of RCE is to design a simple scaler of capital that scales down capital inflated by Jensen's Inequality to remove its upward bias. And this scaling factor needs to be a function of the degree of convexity of VaR for a given severity, its parameter values, the percentile being estimated, and the sample size of the loss sample. The more apparently convex is VaR, the greater the downward scaling required to achieve an expected capital value centered on true capital. Both the magnitude of capital bias and the degree of presumed convexity of VaR, reflected in part in RCE's "*c*" parameter, are functions of i) the severity selected, ii) its estimated parameter values, and iii) the size of the quantile being estimated (e.g. for RCap vs. ECap). The magnitude of capital bias, although not the degree of presumed VaR convexity, also is a function of iv) the sample size of the loss dataset, as shown in Table 2 and Figure 1). However, in its current state of development, *c* is a function only of the severity selected and sample size, which appear to be the dominant drivers of capital bias. As shown in the Results section below, when using only sample size and the severity selected, RCE performs i) extremely well in terms of capital accuracy, eliminating virtually all capital bias except for a few cases under the smallest sample sizes $n \approx 150$, or $\lambda = 15$, ii) notably well in terms of capital precision, outperforming MLE by very wide and consistent margins, and iii) consistently, if less dramatically better than MLE in terms of capital robustness. If the size of the quantile mattered, we would see large differences, for a given value of c, in RCE's capital accuracy (and precision and robustness) for RCap vs. ECap, but that is not the case: there is negligible to very little difference (except for a few cases under the smallest sample sizes of $n \approx 150$, or $\lambda = 15$). Similarly for the parameter values: for a given value of c, but very different parameter values of the same severity, we would expect to see large differences in RCE's capital accuracy (and precision and robustness), but we do not: RCE's capital accuracy (and precision and robustness) is very similar across almost all parameter values of the same severity for a given value of c.

So while derivation of a fully analytic (yet practical) solution to estimating the degree of VaR's presumed convexity that relies on all four inputs may be very desirable, especially if it effectively addresses the few smaller-sample cases where RCE is not completely unbiased (although still much more accurate than MLE), it does not appear to be immediately essential: RCE effectively addresses MLE's deficiencies in terms of capital accuracy and capital precision, and to a lesser degree capital robustness, without identifiable areas in need of major improvements. So this analytic formula, if even possible to derive in tractable form,[43] is left for future research.

Finally, it is very important to note that all four of these inputs, i) – iv), and particularly the two currently used (i.e. the selected severity and sample size), are known ex ante, consistent with capital estimation under the LDA framework, and so they can be used as inputs to estimating capital using RCE without violating the ex ante nature of the estimation.

<u>RCE Conceptually Defined</u>: RCE is conceptually defined below in four straightforward steps.

<u>Step 1</u>: Estimate LDA-based capital using the chosen method (e.g. MLE).

<u>Step 2</u>: Use the severity (and frequency) parameter estimates from Step 1, treating them as reflecting the "true" data generating process, and simulate K data samples and estimate the severity (and frequency) parameter estimates of each.

[43] Note that for their fragility heuristic, a convexity metric in much simpler form than RCE and discussed later in this paper, Taleb et al. (2012) state: "Of course, ideally, losses would be derived in a closed-form expression that would allow the stress tester to trace out the complete arc of losses as a function of the state variables, but it is exceedingly unlikely that such a closed-form expression could be tractably derived, hence, the need for the simplifying heuristic." The excellent performance of RCE presented below in the Results section makes the need to derive undoubtedly complex, closed-form expressions for it much less pressing, or arguably even very useful, with the possible exception of its use under conditions of smaller sample sizes, as discussed below.

Step 3: For each of the *K* samples in Step 2, simulate *M* data samples using the estimated severity (and frequency) parameters as the data generation process, then estimate capital for each of the *M* data samples, and calculate the median of the *M* capital estimates, yielding *K* medians of capital.

Step 4: Calculate the median of the *K* medians of capital, calculate the mean of the *K* medians of capital, and multiply the median of medians by the ratio of the two (median over mean) raised to the power "*c*":

$$\text{RCE} = \text{Median}\,(K \text{ capital medians}) * \left[\text{Median}\,(K \text{ capital medians}) \;/\; \text{Mean}\,(K \text{ capital medians})\right]^{c(sev,n)} \quad (3)$$

The first term of (3) can be viewed as close to the value that would be obtained using Step 1 alone, but it is more stable and thus, is preferable as it contributes to the stability of RCE. The second term – the ratio of median over mean – can be viewed as a measure of the apparent convexity of VaR, because the *K* medians provide a stable trace of the VaR curve (g( ) in Figure 1), so the ratio of median to mean will decrease below 1.0 (one) as the apparent convexity of VaR increases. This ratio is augmented by $c(sev,n)$, which is a function of the sample size and the severity selected. Values for $c(sev,n)$ can be determined in one of two ways: i) using the values provided in Table E1 of Appendix E, by severity by sample size, or ii) using straightforward simulation study on a case by case basis (as was done to obtain the values in Table E1). Both alternatives are discussed in more detail in the following section.[44]

So conceptually, RCE traces the VaR curve shown in Figure 1, and then uses a simple measure of its presumed convexity to scale down the capital estimate. The goal is to scale the right amount so that on average, on the *Y* axis (i.e. capital estimates), J.I. = $E[g(\hat{\beta})] - g(E[\hat{\beta}]) \approx 0$, or slightly above zero to be conservative. This is conceptually straightforward, but simulations of simulations (Steps 2 and 3) can be runtime prohibitive, depending on the sample size and number of UoMs for which capital must be estimated. In the implementation section below, I present a sampling method (actually, a perturbation method) that speeds this effort by orders of magnitude and provides even better stability than simple random sampling, especially for UoMs with smaller sample sizes.

[44] It should be noted here that because the first term of RCE is very close to capital based solely on, say, MLE, in a sense RCE can be viewed as an overlay, calibration, or adjustment to MLE-based capital. If they chose to swap this first term for LDA-based capital, banks currently using LDA would not have to change anything else in their framework to use RCE other than to apply RCE after Step 1 above. This change would alter the values of $c(sev,n)$ slightly, but this (re)calibration would not be onerous as it is identical to that described below: only with LDA-based capital as the first term of (3). The only disadvantage to this swap would be a slight increase in the variance of the capital estimate. Either way, this flexibility is a big advantage of RCE, especially for larger banks with more data, numerous UoMs, and large frameworks already implemented. Their use of RCE could merely be "on top of" their current framework: nothing else would need to change.

RCE Implemented

Step 1: Estimate LDA-based capital using the chosen method (e.g. MLE)

Step 2: Iso-Density Sampling **–** Use the severity parameter estimates from Step 1, treating them as reflecting the "true" data generating process, and invert their Fisher information to obtain their (asymptotic) variance-covariance matrix.[45] Then simply select 4 * K pairs of severity parameter estimates based on selected quantiles of the joint distribution of the severity parameters (those used in this paper correspond to the following percentiles: 1, 10, 25, 50, 75, 90, 99, so $K = 7$). Each severity parameter of the pair is incremented or decremented the same number of standard deviations away from the original estimates, in four directions tracing out two lines of severity parameter values as shown below in Figure 2 (the lines are orthogonal when the parameters are scaled by their standard deviations). In other words, taking the 99%tile as an example, i) both severity parameters are increased by the same number of standard deviations until the quantile corresponding to the 99%tile is reached; ii) both severity parameters are decreased by the same number of standard deviations until the quantile corresponding to the 99%tile is reached; iii) one severity parameter is increased while the other is decreased by the same number of standard deviations until the quantile corresponding to the 99%tile is reached; and iv) one severity parameter is decreased while the other is increased by the same number of standard deviations until the quantile corresponding to the 99%tile is reached. So we now have $K = 7 * 4 = 28$ pairs of severity parameter values. But we must also account for variation in λ, the frequency parameter, and so two values of λ are used in this study: those corresponding to the 25%tile and the 75%tile of the Poisson distribution implied by the original estimate of λ. So now $K = 28 * 2 = 56$.

Step 3: Iso-Density Sampling **–** Using each of the K severity (and frequency) parameter estimates from Step 2 as defining the data generating process, generate via iso-density sampling $M = 7 * 4 * 2 = 56$ new severity (and frequency) parameter values for each set of estimates from Step 2, and now calculate their corresponding capital values. Then calculate the median of these M capital values to end up with $K = 56$ medians of capital.

Step 4: Using the K medians obtained from Step 3, calculate the median and calculate the weighted mean,[46] and multiply the median of medians by the ratio of the two (median over mean) raised to the power "c":

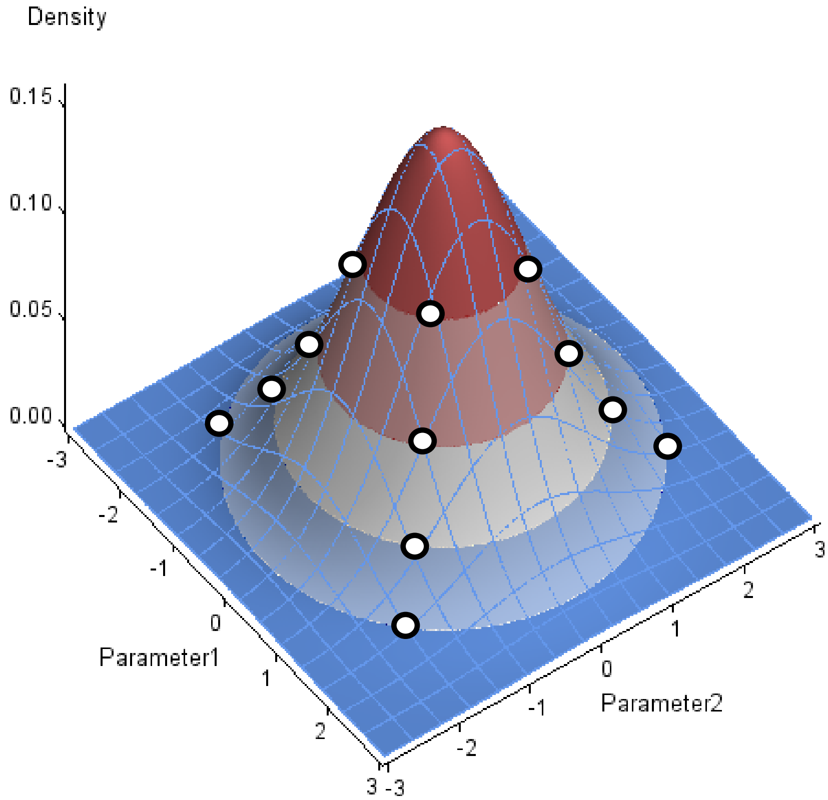
---

[45] Note that for many, if not most estimators used in this setting (e.g. M-class estimators), the joint distribution of the severity parameter estimates will be multivariate normal, and so the initial estimates taken together with the variance-covariance matrix completely define the estimated joint distribution.

[46] Because this is a weighted sampling, the mean is weighted by one minus the percentile associated with a particular iso-density multiplied by two times one minus that associated with the frequency percentile (since the frequency and severity parameters are assumed to be independent – see Ergashev, 2008, for more on this topic; weight = [1 – p-sev] * 2 * [1 – p-freq]). Technically the weighted median should be used alongside the weighted mean, but empirically the weighted median, which requires additional computational steps, always is identical to the unweighted median here due to the symmetry of the joint parameter distribution. And so for efficiency's sake, the unweighted median is used here.

$$\text{RCE} = \text{Median}\ (K \text{ capital medians}) * [\text{Median}\ (K \text{ capital medians})\ /\ \text{Mean}\ (K \text{ capital medians})]^{c(sev,n)} \quad (3)$$

FIGURE 2: Iso-density Sampling of the Joint Severity Parameter Distribution



This is a rapid and stable way to systematically perturb parameters,[47] based on the joint (asymptotic) parameter distribution, to obtain a view of capital as a function of VaR. It also is quite accurate, arguably even more accurate for smaller samples than relying on simple random sampling, which for some of the data samples and some of the parameter values not uncommon in this setting can lead to truly enormous empirical variability. Asymptotically, in theory, both approaches are approximately equivalent as long as proper weighting is used when sampling via iso-densities. But in practice, simple random sampling in this setting can be i) extremely variable and unstable; ii) often more prone to enormous data outliers than theory would lead one to expect; and iii) often more prone to enormous estimate outliers because for many heavy-tailed severities, estimation of large parameter values simply is very difficult and algorithmic convergence is not always achieved. Even though iso-density "sampling" (perturbation) relies on an asymptotic result, it appears to not only be a much faster alternative, but also a more stable one in this setting, which is characterized by smallish samples and extremely skewed, heavy-tailed densities (not to mention heterogeneous loss data even within UoMs).

[47] Note that the point of iso-density "sampling" is *not* to draw a representative sample of the joint parameter distribution, but rather, to systematically perturb parameters for the purpose of tracing VaR as a function of the severity parameters.

To efficiently obtain the values of the severity parameters on a specified percentile ellipse,[48] one must utilize knowledge of the joint parameter distribution, or at least its approximation. If using, say, any M-class estimator to estimate severity parameters, we know the joint (asymptotic) distribution of the estimates is multivariate normal. With knowledge of the Fisher information of each,[49] therefore, we can use (4), where *x* is a *k*-dimensional vector of parameter values (incremented/decremented away from μ by the same number of standard deviations), μ is the known *k* -dimensional mean vector (the parameter estimates), $\sum$ is the known covariance matrix (the inverse of the Fisher information of the given severity), and $\chi_k^2(p)$ is the quantile function for probability *p* of the Chi-square distribution with *k* degrees of freedom.

$$(x-\mu)^T \Sigma^{-1}(x-\mu) \leq \chi_k^2(p) \qquad (4)$$

In two-dimensional space, i.e. when $k = 2$, which is relevant for the widespread use of two-parameter severities in this setting, this defines the interior of an ellipse, which is a circle if there exists no dependence between the two severity parameters (if the joint parameter distribution is multivariate normal, a circle will be defined if the (Pearson's) correlation is zero). *x* represents the distance from the parameter estimates, μ. Thus can one find the values of the severity parameters that provide a specified quantile of the joint distribution with (4). One can chose points on the ellipse that correspond to movement of each parameter the same number of standard deviations away from μ by using (5). Simply increment/decrement both parameters by *q* units of their respective standard deviations to obtain four pairs on the ellipse: increase both parameters by *q* standard deviations $(z_1 = z_2 = 1)$, decrease both parameters by *q* standard deviations $(z_1 = z_2 = -1)$, increase one while decreasing the other $(z_1 = 1, z_2 = -1)$, and decrease one while increasing the other $(z_1 = -1, z_2 = 1)$.

$$q\#stdev = \sqrt{\frac{\chi_k^2(p)\cdot(1+z_1z_2\rho_{1,2})}{2}} \qquad (5)$$

where $\sigma_1(\sigma_2) =$ stdev of parameter 1 (2), and $\rho_{1,2}$ is the correlation between the parameter estimates. (see Mayorov, 2014). Alternately, the eigenvalues and eigenvectors of Σ^{-1} can be used to define the most extreme parameter values (smallest and largest) on the ellipses (corresponding to the largest/smallest eigenvalues) (see Johnson and Wichern, 2007), but this would likely change the values of *c*(*sev*,*n*) calculated in Appendix E, and (5) is arguably more straightforward.

[48] The specified percentile represents the percentage of the joint density within the ellipse. For severities with more than two parameters, and so dimensions higher than two, this is termed an ellipsoid.

[49] See Appendix D.

Other approaches to mitigating bias due to convexity, typically using bootstraps or exact bootstraps to shift the distribution of the estimator, simply do not appear to work in this setting either because the severity quantile that needs to be estimated is so extremely large (e.g. $[1 - (1\text{-}\alpha)/\lambda] = 0.99999$ for ECap assuming λ=30), or because this quantile is extrapolated so far "out-of-sample," or because VaR is the risk metric that must be used, or some combination of these reasons (see Kim and Hardy, 2007). Some that were tested here worked well for a particular severity for a very specific range of parameter values, but in the end all other options failed when applied across very different severities and very different sample sizes and very different parameter values. RCE was the only approach that reliably estimated capital unambiguously better than did MLE under the LDA framework,[50] across the wide range of conditions examined in this paper (see Simulation Study section below).

An important implementation note must be mentioned here: when calculating capital based on large severity parameter values, say, the 99%tile of the joint distribution in Step 3, based on 99%tile severity estimates generated in Step 2, based on (sometimes) already large estimate of severity parameters originally, sometimes capital becomes incalculable: in this example, the number simply is too large to estimate (in this study, this only occurred, and rarely, for the severities with the heaviest tails: TGPD and TLOGG). So we need to ensure that missing estimates do not cause bias: for example, that a scenario cannot occur whereby only the "decrease, decrease" arm of the iso-density sample in Figure 2 has no missing values. Therefore, if any capital values are incalculably large on an ellipse, the entire ellipse, and all ellipses "greater" than it, are discarded from the calculation. This ensures that the necessary exclusion of incalculably large capital numbers do not bias statistics calculated on the remaining values, which by definition are symmetric around the original estimates.

Finally, I address here how $c(sev, n)$ is defined and calculated. Table E1 in Appendix E presents values of $c(sev, n)$ by severity by sample size which were empirically determined via simulation studies. The simulation study simply generated 1,000 RCE capital estimates for a given sample size for a given severity for different values of c: the value of c that came closest to being unbiased, with a slightly conservative leaning toward small positive bias, is the value of c used. Sample sizes tested, for a ten year period, included average number of loss events = λ = 15, 25, 50, 75, and 100 for samples of approximately $n \approx 150$, 250, 500, 750, and 1,000 loss events.[51] This is a very wide range of sample sizes compared to those examined in the relevant literature (see Ergashev et al, 2014, Opdyke and Cavallo, 2012a and 2012b, and Joris, 2013), and it arguably covers the lion's share of sample sizes in practice, unfortunately with the exception of the very small UoM's. For all sample

[50] Again, "better" here means with greater capital accuracy, greater capital precision, and greater capital robustness.

[51] As described previously, a Poisson frequency distribution was assumed, as is widespread accepted practice in the industry. Sample sizes are approximate because they are a function of a random variable, λ. This is described in more detail below.

sizes in between, from 150 to 1,000, straightforward linear and non-linear interpolation is used, as shown in Figure E1 in Appendix E, and preliminary tests show this interpolation to be reasonably accurate.[52] The Results section describes in detail the effects of sample size (and severity selected) on RCE-based capital estimates.

The second way to obtain and use values of *c*(*sev*, *n*) is to simply conduct the above simulation study for a specific sample size and, say, three sets of severity parameters: the estimated pair (for a two-parameter severity), a pair at the 2.5%tile of the joint parameter distribution (obtained from (4)), and a pair at the 97.5%tile to provide a 95% joint confidence internal around the estimated values. If the same value of *c*(*sev*, *n*) "works" for all three pairs of severity values,[53] thus appropriately taking into account severity parameter variability, then it is the right value for "*c*." As described in the Results section below, the distribution of RCE-based capital estimates was surprisingly robust to varying values of *c*(*sev*, *n*). In other words, the same value of *c*(*sev*, *n*) "worked" for very large changes in severity parameters (and capital), and very large changes in percentiles (e.g. RCap vs. ECap). So determining the value of *c*(*sev*, *n*) empirically in this way, i.e. testing to make certain that the same value of *c*(*sev*, *n*) holds for ±95% joint confidence interval (or a wider interval if deemed more appropriate), should properly account for the fact that our original severity parameter estimates are just that: inherently variable estimates of true and unobservable population values. All sample sizes beyond the range examined in this paper (i.e. $n < 150$ or $n > 1{,}000$) should make use of this approach, although for some severities, *c*(*sev*, *n*) may vary by parameter values for small *n*. So caution is urged in the application of RCE to smaller samples than studied in this paper. Note again from Table 2 that for larger sample sizes beyond $n \approx 1{,}000$, all severities will exhibit much less bias because parameter variance is sufficiently small. However, RCE is extremely useful even in these cases in notably reducing capital variability and increasing model stability, as discussed further below.

Before addressing RCE runtimes below, I describe two more innovations, in addition to the efficient use of iso-density sampling, that are derived in this paper and that increase runtime speed by nearly an order of magnitude for one of the severities examined (a fourth innovation related to both runtime speed and extreme quantile approximation is presented in the next section). The two-parameter Truncated LogGamma distribution typically is parameterized in one of two ways: either with its second shape parameter, b, specified as b, or alternatively, 1/b. The latter is used throughout this paper. An analytic expression of the mean of the former is

[52] The non-linear interpolation is based on (6) presented in the next section.

[53] Here, "works" means that the three means of each of the three capital distributions of 1,000 RCE capital estimates all are very close to their respective "true" capital values.

provided in Kim (2010),[54] but a corresponding result for the latter does not appear to have been derived in the literature, so this is done in Appendix D. Also, while the Fisher information of the Truncated LogGamma has been derived and used for operational risk capital estimation previously (see Opdyke and Cavallo, 2012a, and for the non-inverted parameterization of the Truncated LogGamma, see Zhou, 2013), both examples rely on computationally expensive numeric integration, so an analytic approximation is derived and presented in Appendix D that provides speed increases, for very precise approximations,[55] of between seven and ten times faster than that required for numeric integration.

Relying on these innovations, RCE runtimes are shown below in Table 3 by severity by sample size. All analyses presented in this paper were conducted using SAS® on a desktop computer (16GB RAM, 64-bit OS, CPU @ 3.40GHz). Note that because of the efficient use of the Fisher information and iso-density sampling, sample size has no effect on runtimes. Runtimes only vary by severity based on the complexity of the Fisher information and the capital calculation. Most importantly, note that to estimate capital for one UoM, implementing RCE as described above takes only a second or two, even for severities with the most complex Fisher informations (e.g. the Truncated LogGamma).

TABLE 3: Runtime Speed of RCE by Severity (seconds)

	Non-Truncated		Truncated (*H*=$10k)	
Severity	**Real Time**	**CPU Time**	**Real Time**	**CPU Time**
LogNormal	0.14	0.14	1.10	1.10
LogGamma	1.13	1.12	2.96	2.94
GPD	0.21	0.18	1.35	1.35

From the above description of RCE, it should be clear that the general approach taken here to developing a capital estimator that is straightforward to understand, easy to implement, and *that works in practice* has been one of reliance on both appropriate theoretical results as well as practical empiricism. This is consistent with the late George Box's[56] approach to the development and use of robust methods. Box emphasized that "practical need often leads to theoretical development" (Box, 1984) and that "to obtain a useful procedure, one needs *both empiricism and theory*. But – more than that – one needs continuous iteration between them…" (Box and Luceño, 1998). This iteration not only guided the development of RCE specifically, and should guide

[54] Technically, Kim (2010) provides the conditional tail expectation, which is different from the truncated mean, although the former simply needs to be multiplied by a constant to obtain the latter. See Appendix D for more details.

[55] The largest deviations from true capital when using this approximation in all the simulation studies conducted herein were a few thousand dollars when true capital was in the hundreds of millions, and sometimes billions of dollars.

[56] George Box is widely recognized as one of the fathers of modern, applied statistics. He often is cited as the source of the adage, "All models are wrong, but some models are useful."

its further development, but also should guide research on this topic area generally as increased focus is placed on the capital distribution. As operational risk modelers know, operational risk capital estimation is nothing if not highly resistant to idealized, textbook solutions that are mathematically convenient and neatly packaged. Keeping an open mind to pursuing approaches that appropriately balance important theoretical underpinnings with practical empirical, methodological, and regulatory constraints to arrive at workable, useable solutions for real world problems is absolutely necessary in this setting, and must drive the research agenda beyond strictly theoretical work that has limited use in practice.

RCE Range of Application: Severity Estimators

Although RCE can make use of most, if not all severity estimators used in this setting, it is implemented here essentially as an overlay or calibration to MLE for several reasons. Not only is MLE typically the fastest and easiest estimator to implement, but also it is most appropriate here for comparability purposes: we must hold all else constant when comparing RCE to the most widely used alternative, that is, MLE. Then the comparison is truly "apples-to-apples," because we know the only difference between the two capital distributions of RCE (when based on MLE) vs. MLE (used alone) is the use of RCE, and any observed differences are due only to RCE and cannot be the result of using some other estimator in Step 1 of RCE's implementation.

RCE Range of Application: Methods of Extreme Quantile Estimation

Another innovation presented in this paper is an improved method for approximating the extreme quantile of the aggregate loss distribution when the severity distribution can be characterized by infinite mean, *or close*.[57] As discussed above, the convolution of the frequency and severity distributions rarely yields a closed-form aggregate loss distribution from which VaR is easily estimated, but a number of methods for approximating VaR are widely used. These include mean-adjusted Single Loss Approximation (SLA, see Degen, 2010), Fast Fourier Transform (FFT, see Embrechts and Frei, 2009), Panjer Recursion (see Panjer, 1981, and Embrechts and Frei, 2009), extensive Monte Carlo simulation (see Opdyke and Cavallo, 2012a), numeric approximations (e.g. the Direct Method – see Kato, 2013), Indirect Estimation (see Sahay et al., 2007), and Closed-Form Approximations (see Hernandez et al., 2013). Extensive Monte Carlo simulation is the gold standard here, but it remains extremely computationally expensive because the quantiles being estimated are so large, so very large numbers of simulations are required to adequately represent the extreme empirical tail of the loss distribution. FFT is stable and faster than Panjer recursion, but mean-adjusted SLA is the most widely used method as it has the advantage of being a straightforward formula. As such it provides the fastest

[57] "Or close" is emphasized here because any method that relies on simulation, as does RCE, will need to work under conditions of infinite mean upon encountering parameter values that are *close* to values that induce an infinite mean, because the simulations based on them will invariably generate some parameter values that correspond to infinite mean severities.

implementation, especially when testing or comparing estimators which typically requires many simulations. SLA also is widely accepted as being sufficiently accurate (see Hess, 2011; and Opdyke and Cavallo, 2012a). But it suffers from a serious implementation flaw: it contains divergent roots when, for severities that can have means approaching infinity (herein, for example, GPD, LogGamma, Truncated GPD, and Truncated LogGamma), the tail index approaches the value "one," either from above or below.[58] This is shown in Figure 3 for GPD, with reference to (2.a) and (2.c), below and above $\xi = 1$, respectively. Importantly, the parameter values do not need to be very close to "one" for the capital approximation to noticeably diverge from its true value,[59] so something must be done to address this when relying on data that is fitted to these distributions (or relying on data simulated from these distributions) to estimate capital. While the indirect method of Sahay et al. avoids this problem, it requires an additional loop for a root-finding algorithm, and so most likely is slower than a formula-based approach.

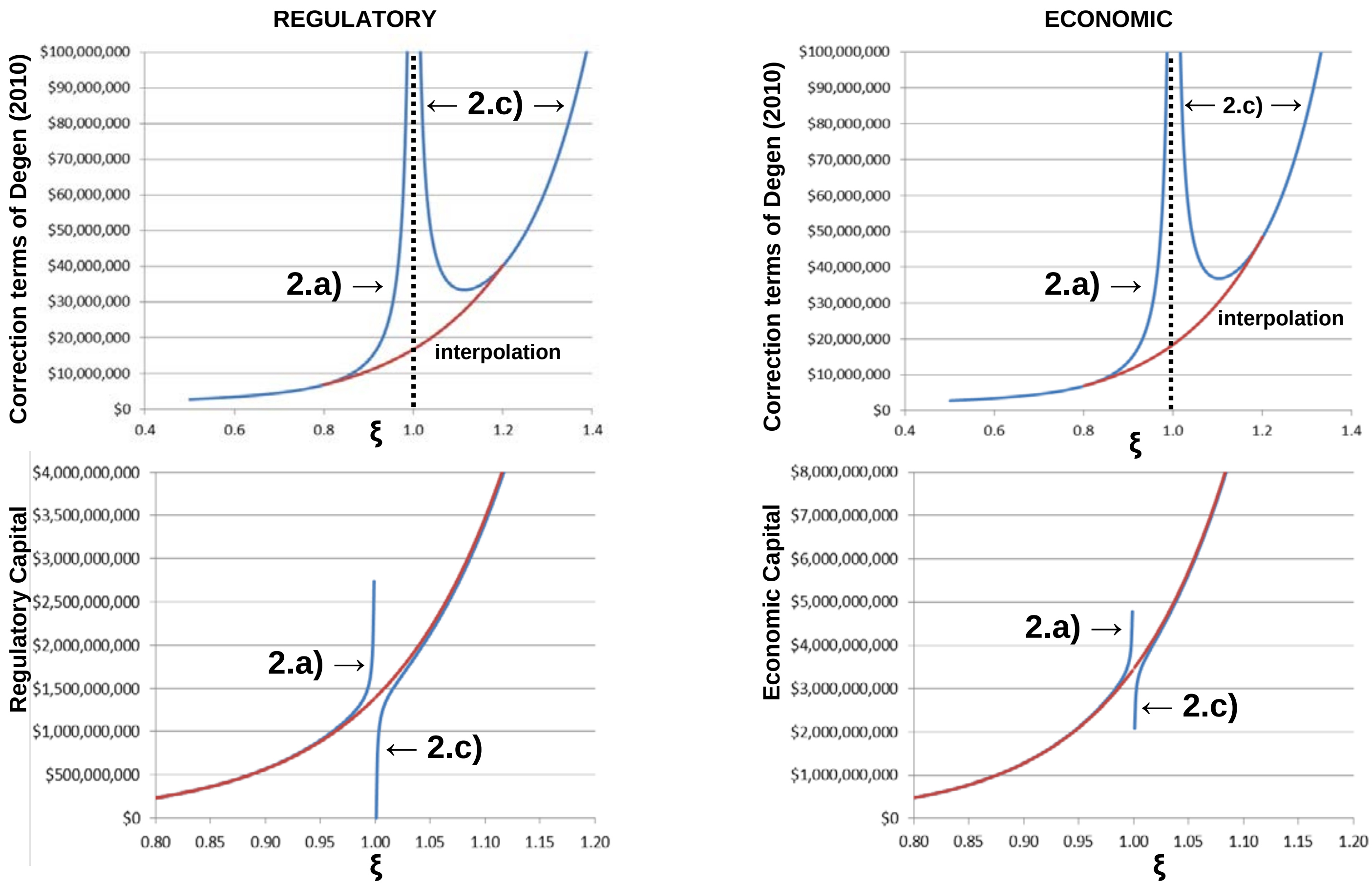
Both the MLE and RCE implementations of LDA in this paper make use of "ISLA" – Interpolated SLA – to avoid divergence in the capital estimate as tail index values approach one, yet still retain the speed advantages of a formula-based approximation. ISLA uses a straightforward nonlinear interpolation at predefined starting and ending points of the tail index values, as shown in (6) below. All notation corresponds to that used in (2.a,b,c).

Both the precision value (PRE = 1,000) and the root value (Root = 50) were sufficiently accurate in this setting: estimated capital via ISLA always was within ±1% of estimated capital based on extensive monte carlo simulation for all severities where infinite means are possible (herein, LogGamma, Truncated LogGamma, GPD, and Truncated GPD). The example of GPD is shown in Figure 3. Capital is calculated for $\xi = 0.8$ and $\xi = 1.2$ (using the estimated value for θ), and if the estimated ξ lies within this range, the interpolated capital value is used. This straightforward, if brute force method provides very accurate approximations to true capital, and importantly, is very fast computationally: ISLA is almost an order of magnitude faster than root-avoiding alternatives that require numeric integration (see Mignola and Opdyke, 2012). Of course, the start and endpoints for the interpolation must be determined for each severity, and sometimes can vary slightly based on

[58] Verifying whether the Closed-Form Approximations of Hernandez et al. (2013) avoid this root divergence is beyond the scope of this paper.

[59] This divergence holds regardless of sample size: it does not disappear asymptotically, that is, even if the number of losses in the sample approaches infinity. The divergence below $\xi = 1$ is due to the mean approaching ∞ (see 2.a), and divergence above $\xi = 1$ is due to the fact that $\Gamma(s)$ diverges as s approaches zero from either direction, so $\Gamma(1 - 1/\xi)$ will diverge as ξ approaches one (see 2.c).

FIGURE 3: Correction for SLA Divergence at Root of $\xi = 1$ for GPD Severity ($\theta = 55{,}000$)



$$\text{Low Value of Tail Parameter} = \text{LTP} = \xi_{LOW} \quad (= 0.80 \text{ for GPD})$$

$$\text{High Value of Tail Parameter} = \text{HTP} = \xi_{HIGH} \quad (= 1.20 \text{ for GPD})$$

$$\text{High Correction Term} = \text{HCT} = (1-\alpha)F^{-1}\left(1-\frac{1-\alpha}{\lambda};\xi_{HIGH}\right)\cdot\left(\frac{c_{\xi_{HIGH}}}{1-1/\xi_{HIGH}}\right) \quad \text{where } c_{\xi_{HIGH}} = (1-\xi_{HIGH})\frac{\Gamma^2(1-1/\xi_{HIGH})}{2\Gamma(1-2/\xi_{HIGH})}$$

$$\text{Low Correction Term} = \text{LCT} = \lambda\mu_{LOW} \quad (\text{where } \mu_{LOW} \text{ is based on } \xi_{LOW})$$

$$\text{Precision} = \text{PRE} = 1{,}000\,; \quad \text{Root} = 50$$

$$\text{Full Range Count} = \text{FRC} = (\xi_{HIGH} - \xi_{LOW})\cdot \text{PRE}$$

$$\text{Count in Range} = \text{CIR} = (\xi - \xi_{LOW})\cdot \text{PRE} \quad \text{where } \xi \text{ is estimated from the sample}$$

$$\text{Difference Root Scale} = \text{DRS} = [\text{HCT}^{(1/\text{ROOT})} - \text{LCT}^{(1/\text{ROOT})}] / [\text{FRC} - 1]$$

$$\text{Interpolated Correction Term} = \text{ICT} = [\text{LCT}^{(1/\text{ROOT})} + \text{CIR}\cdot\text{DRS}]^{\text{ROOT}}$$

$$\text{Estimated Capital via ISLA} = \approx F^{-1}\left(1-\frac{1-\alpha}{\lambda}\right) + \text{ICT} \qquad (6)$$

the parameter values of the distribution.[60] But determining these values once, visually, is straightforward, as shown in Figure 3, and given the conditional nature of LDA capital estimation (i.e. it is conditioned upon the selection of a severity distribution first), this does not violate the ex ante nature of the estimation process.

Simulation Study

Framework

This simulation study compares i) LDA-based capital estimates relying solely on MLE for severity and frequency parameter estimation (the most common implementation of LDA) versus ii) LDA-based capital estimates generated by RCE implemented as described in the previous section (where MLE is used for the frequency and severity parameter estimation). One thousand samples of loss data, which are i.i.d. in the basecase, are simulated to generate one thousand capital estimates, and the characteristics of the two capital distributions are compared to each other, and to the true capital values. As described above, the three main criteria examined are capital accuracy (unbiasedness), capital precision (the spread of the capital distribution), and capital robustness (i.e. distribution characteristics under contaminated, non-i.i.d. data). Accuracy is determined by the capital bias: simply, are the expected values of the capital distributions close to, or far from the true capital values? Spread is determined by a number of descriptive statistics, including the standard deviation, inter-quartile range, coefficient of variation, 95% empirical confidence intervals, and RMSE (which also incorporates bias) of the two capital distributions. Other important distributional characteristics, such as the skewness and kurtosis, also are compared. Finally, robustness is determined by an MLE vs. RCE comparison of deviations from the two respective capital distributions under i.i.d. simulations when non-i.i.d. data simulations are generated, as described below.

It is very important to note that by both design and necessity, the results from this simulation study assume that the right model, i.e. the right severity (but of course, not static parameter values), is selected to estimate capital: the choice of severity in this setting has notoriously low statistical power, and is yet another source of enormous variance in capital estimation. However, severity selection is a distinct component of the capital estimation framework, and by design this study focuses on capital estimation conditional on (the right) severity selection, just as the LDA framework does. To compare capital estimators, by necessity all else must be held constant, so this study does not examine and include other sources of variance in LDA-based capital from other areas of the framework (such as severity selection or estimating dependence structure across UoMs).

[60] This is true for the LogGamma and Truncated LogGamma, but values of "b" do not approach "one" in this setting, so this is more a statistical coding precaution than an immediate concern for capital calculation, as it is for GPD (and Truncated GPD) as shown in Figure 3. Note, too, that only for the GPD does the tail index happen to equal ξ: for the LogGamma, for example, the tail index = 1/b.

Sample Size

The basecase sample size of losses ($n \approx 250$, corresponding to ten years of losses with $\lambda = 25$) is conservatively set a bit larger than that of many UoM's so that any observed effects of Jensen's inequality on the MLE-based capital numbers generally are understated relative to the "typical" UoM (because in relative terms, the effects almost always increase as sample sizes decrease). Table 2 above shows four additional simulations of the MLE-based capital estimates for smaller ($n \approx 150$) and larger ($n \approx 500$, 750, and 1,000) samples to demonstrate empirically the very strong role that the size of the variance of the severity parameter estimators, vis-à-vis sample size, plays in biasing capital estimates.[61]

Severity (and Frequency) Distributions

Operational risk losses are simulated based on a Poisson frequency distribution, and six of the most commonly used severity distributions:[62] the LogNormal, LogGamma, and the Generalized Pareto (GPD) distributions,[63] as well as the truncated versions of each with a truncation threshold of H = \$10,000, arguably the most widely used data collection threshold. As described above, the use of truncated distributions is the most widely accepted method for addressing data collection thresholds, unlike some alternatives that have been pillared in regulatory review processes and in the literature (for example, the use of so-called "shifted" distributions – see Schevchenko, 2009, and Ergashev et al., 2014; but for a counter-argument supporting shifted distributions, see Cavallo et al., 2012).

As described above, the implementation of RCE efficiently utilizes the Fisher information, and analytic derivations exist for five of the six severities listed above (see Appendix D, which includes a) definitions of these severities, b) closed-form analytic expressions for their means for use in calculating capital, and c) their Fisher informations). For the Truncated LogGamma, a new analytic approximation of the Fisher information is derived in Appendix D, thus avoiding computationally costly numeric integration. This speeds computer runtime by almost an order of magnitude, so this approximation is used for this severity for both MLE and RCE-based capital estimates. Also derived in Appendix D is a closed-form analytic expression for the mean of

---

[61] As mentioned previously, these findings are exactly consistent with other empirical results in the literature (for example, see Ergashev et al., 2014, and Opdyke and Cavallo, 2012a).

[62] See for example, Opdyke, 2013, Opdyke and Cavallo, 2012a, 2012b, Zhou et al., 2013, and Joris, 2013.

[63] These are three of the four parametric severity distributions listed in the most recent Interagency Guidance on Operational Risk AMA severity estimation (see OCC, 2014).

the Truncated LogGamma severity,[64] which also decreases runtimes over the alternative requiring numeric integration.

Range of Parameter and Capital Values

Severity parameter values for simulating capital were selected i) to reflect values commonly cited in the literature (see Opdyke and Cavallo, 2012a, 2012b; Joris, 2013, and Zhou, 2013); ii) to reflect a very wide range of capital values (from $38m to over $10.6b) to better represent the wide range of conditions under which capital inflation is slightly vs. notably vs. extremely large (sometimes well over 100% greater than true capital), and to fully test the behavior of RCE; and iii) to reflect different parameter values (of the same severity) while holding capital roughly constant to demonstrate the different effects of individual severity parameters on capital, and how these effects are exactly consistent with Jensen's inequality. The basecase is $\lambda = 25$ for ten years of data, or approximately $n \approx 250$. Parameter values generate (true) regulatory capital in the basecase that ranges from $53m to over $2.7b. While some may consider the upper end of this range large at the level of the UoM, the bulk of the range actually is fairly conservative for many large banks, and even many regional or mid-sized banks. For some of the larger sample sizes, however, such as $\lambda = 75$ and 100, even though the same parameter values are used as with the smaller sample sizes, the larger numbers of losses generate capital numbers that stretch the bounds of what would be seen in practice. Again, the purpose for this was to test the bounds of RCE, as well as to determine the point at which sample sizes increased enough to notably mitigate the upward bias of LDA-MLE-based capital. Otherwise, focus should be on the capital numbers corresponding to $\lambda = 25$, which despite their size, correspond to parameter values that are not uncommon in practice, especially for the larger banks.

Method of Capital Calculation / Approximation

As defined above, the ISLA method is used for approximating the VaR of the aggregate loss distribution to estimate capital, both for RCE and MLE. To calculate (2.a), analytic formulae for the means of all the severities used in this study are presented in Appendix D.

Robustness

Robustness of MLE vs. RCE is tested via "left tail," "right tail," and "both tail" contamination of the i.i.d. basecase sample. The left tail distribution is the same severity as the basecase, but with different parameter values that correspond to the lower 5%tile of the joint parameter distribution (where both basecase parameter

---

[64] Note that Kim (2010) presents an analytic expression for the mean of an alternate parameterization of the truncated LogGamma: his is based on a Gamma distribution with two shape parameters, rather than a shape parameter and an inverted (b=1/b) shape parameter, as is used in this paper.

values are decreased[65] by the same number of standard deviations via (5) to obtain these values). The same is true for the right tail distribution, but with parameter values that both are increased to correspond to the upper 95%tile. This conceptually is the equivalent of a multivariate 90% confidence interval, which is a very plausible, if not conservative reflection of a non-textbook, non-i.i.d. empirical reality (alternately, Johnson and Wichern, 2007, can be used to obtain parameter values on the same ellipse corresponding to the largest eigenvalues). Each contaminating distribution comprises 5%, on average, of the overall severity (and so the "both tails" case has 5% contamination from each tail, on average (because the percent of the distribution contaminated is stochastic)).

While the above addresses violations of the "identical distribution" portion of the i.i.d. assumption, incorporating the effects of violations of the independence assumption on operational risk capital estimation has been largely ignored in this setting (the few exceptions include Guégan and Hassani, 2013, Umande, 2013, and Embrechts et al., 2013). This may be due, as least in part, to flexibility on this specific issue in the US Final Rule (2007).[66] While such testing is beyond the scope of this study, operational risk losses are very likely to be serially correlated (see Guégan and Hassani, 2013), and other empirical examinations of this issue generally show that its deleterious effects on statistical inference can be very material (see van Belle, 2008). So this is an issue that should continue to be further addressed in future research.

RCE parameters

Values of $c(sev, n)$ used are those listed in Table E1. Iso-densities used for sampling correspond to the percentiles listed above: 1, 10, 25, 50, 75, 90, and 99 for severity parameters, and 25 and 75 for the frequency parameter. RCE capital is based on severity and frequency parameters estimated using MLE. While RCE can be applied to capital estimated with almost any estimator, as mentioned above the use of MLE makes "all else equal" when comparing RCE capital to capital estimated via the most widely used estimator (MLE). In other words, the only differences must be attributed to RCE. And these differences will have the most direct relevance to the largest number of financial institutions because most currently are using only MLE.

---

[65] The exceptions to this are the LogGamma and Truncated LogGamma distributions. In this paper, they rely on a Gamma parameterization with a shape parameter and an inverted (b=1/b) shape parameter, which consequently means that smaller values of b correspond to larger capital estimates, so b is decreased where all other severity parameters are increased, and vice versa.

[66] See US Final Rule (2007), p.69318: "A bank's chosen unit of measure affects how it should account for dependence. Explicit assumptions regarding dependence across units of measure are always necessary to estimate operational risk exposure at the bank level. *However, explicit assumptions regarding dependence within units of measure are not necessary, and under many circumstances models assume statistical independence within each unit of measure. The use of only a few units of measure increases the need to ensure that dependence within units of measure is suitably reflected in the operational risk exposure estimate.*" (emphasis added).

Results

Results of the simulation study described above can be seen in Tables 4a,b and 5 below, and complete results can be found in Appendix F in Tables F4a,b-F11a,b. Based on the three criteria that arguably are the only ones that matter for assessing the effectiveness of an operational risk capital estimation framework – capital accuracy, capital precision, and capital robustness – the improvements provided by RCE over MLE are, respectively, nothing short of dramatic, very notable, and modest but consistent. Except for discussions relating either specifically to sample size or specifically to robustness, below I focus on Tables 4a,b, which is the basecase of $\lambda = 25$ with number of losses $n \approx 250$ (each of the tables, including Tables 4a,b as well as F4a,b-F8a,b in Appendix F, corresponds to a different sample size related to $\lambda = 25$, 15, 50, 75, and 100, respectively; and "a" and "b" indicate RCap and ECap, respectively. Additionally, Tables F9a,b-11a,b correspond to right-tail contamination, left-tail contamination, and both-tail contamination, respectively).

Capital Accuracy

Empirically, we can see that systemically positive bias in MLE-based capital estimates is driven by the three factors identified above: i) As shown earlier in Table 2, sample size drives parameter estimate variance which drives MLE-based capital bias: larger sample sizes are associated with smaller bias, all else equal (see Table 2, and Tables F4a,b-F8a,b). This is exactly consistent with the findings of Ergashev et al. (2014) and Opdyke and Cavallo (2012a). ii) Higher values of VaR, represented by ECap, always exhibit far more bias than lower values of VaR, represented by RCap, all else equal (see Table 4a vs. 4b). iii) Heavier tailed distributions exhibit more bias, all else equal (LogNormal is least heavy, GPD is heaviest, and truncated distributions exhibit more capital bias than their non-truncated counterparts, all else equal, although by design parameter values vary to some degree for the truncated and non-truncated versions of the same severity in this study to keep the overall range of capital estimates at comparable levels) (see Tables 4a,b).

To be more specific regarding iii), consistent with the results of Appendix A, Figure A1, we must note that for the same severity, capital is roughly the same in each pair of adjacent rows of Tables F4a,b-F8a,b, but MLE-based capital bias is consistently larger for the second row of the pair for the LogNormal distributions and GPD distributions. For the former, VaR is a convex function of both μ and σ, but the biasing effect of the shape parameter, σ, dominates over that of the scale parameter ($\log(\mu)$), so when σ is the larger of the two for roughly the same level of capital, capital bias is larger. For the GPD distributions, VaR is a convex function of ξ, but a linear function of θ, and so the latter induces no biasing effects at all on its own (although it is positively correlated with ξ). So when ξ is the larger of the two for roughly the same level of capital, capital bias is larger. The LogGamma, on the other hand, has two shape parameters which are negatively correlated, and VaR is a convex function of both (see Appendix A, Figure A1). This is why the results are mixed for the LogGamma:

TABLE 4a: RCE vs. MLE RCap Distributions – Bias and RMSE by Severity by Parameter Values ($millions)

Severity				Mean			Mean			RMSE	RMSE	RMSE
Dist.	Parm1	Parm2	True	MLE	MLE	MLE	RCE	RCE	RCE	MLE	RCE	RCap
	μ	σ	RCap	RCap	Bias	Bias%	RCap	Bias	Bias%	RCap	RCap	RCE/MLE
LogN	10	2	$63	$67	**$4**	**6.7%**	$63	$0	**0.5%**	$25	$23	**91.8%**
LogN	7.7	2.55	$53	$59	**$6**	**11.5%**	$54	$1	**1.5%**	$30	$26	**87.7%**
LogN	10.4	2.5	$649	$720	**$72**	**11.0%**	$658	$9	**1.4%**	$355	$313	**88.1%**
LogN	9.27	2.77	$603	$686	**$83**	**13.8%**	$614	$12	**2.0%**	$382	$328	**86.0%**
LogN	10.75	2.7	$2,012	$2,275	**$263**	**13.1%**	$2,048	$37	**1.8%**	$1,229	$1,063	**86.5%**
LogN	9.63	2.97	$1,893	$2,198	**$305**	**16.1%**	$1,939	$46	**2.4%**	$1,329	$1,121	**84.3%**
TLogN	10.2	1.95	$76	$85	**$9**	**11.5%**	$75	-$1	**-1.8%**	$52	$41	**79.4%**
TLogN	9	2.2	$76	$96	**$20**	**26.5%**	$75	-$1	**-1.4%**	$88	$50	**56.9%**
TLogN	10.7	2.385	$670	$847	**$177**	**26.4%**	$700	$30	**4.5%**	$665	$469	**70.5%**
TLogN	9.4	2.65	$643	$894	**$251**	**39.1%**	$628	-$14	**-2.2%**	$1,087	$536	**49.3%**
TLogN	11	2.6	$2,085	$2,651	**$566**	**27.1%**	$2,123	$38	**1.8%**	$2,568	$1,771	**69.0%**
TLogN	10	2.8	$1,956	$2,743	**$787**	**40.2%**	$1,965	$9	**0.5%**	$3,033	$1,694	**55.9%**
	a	**b**										
Logg	24	2.65	$85	$97	**$12**	**13.6%**	$87	$2	**2.1%**	$62	$53	**86.0%**
Logg	33	3.3	$100	$108	**$8**	**8.5%**	$99	$0	**-0.4%**	$56	$50	**89.0%**
Logg	25	2.5	$444	$513	**$70**	**15.7%**	$455	$11	**2.5%**	$355	$301	**84.8%**
Logg	34.5	3.15	$448	$497	**$49**	**10.9%**	$452	$4	**0.8%**	$296	$260	**87.6%**
Logg	25.25	2.45	$766	$906	**$140**	**18.3%**	$799	$32	**4.2%**	$647	$543	**83.9%**
Logg	34.7	3.07	$818	$930	**$112**	**13.7%**	$841	$23	**2.8%**	$589	$510	**86.6%**
TLogg	23.5	2.65	$124	$193	**$70**	**56.1%**	$137	$13	**10.8%**	$273	$181	**66.4%**
TLogg	33	3.3	$130	$174	**$44**	**34.1%**	$130	$0	**-0.1%**	$173	$93	**53.9%**
TLogg	24.5	2.5	$495	$794	**$299**	**60.4%**	$516	$20	**4.1%**	$1,103	$554	**50.2%**
TLogg	34.5	3.15	$510	$635	**$125**	**24.5%**	$539	$29	**5.8%**	$544	$397	**73.1%**
TLogg	24.75	2.45	$801	$1,305	**$504**	**62.9%**	$848	$47	**5.9%**	$1,938	$916	**47.3%**
TLogg	34.6	3.07	$867	$1,078	**$211**	**24.3%**	$925	$58	**6.7%**	$927	$709	**76.5%**
	ξ	**θ**										
GPD	0.8	35,000	$149	$233	**$85**	**56.9%**	$152	$3	**2.2%**	$295	$167	**56.7%**
GPD	0.95	7,500	$121	$212	**$91**	**75.6%**	$124	$3	**2.7%**	$311	$156	**50.3%**
GPD	0.875	47,500	$391	$640	**$249**	**63.7%**	$396	$5	**1.2%**	$870	$466	**53.6%**
GPD	0.95	25,000	$403	$697	**$295**	**73.2%**	$408	$5	**1.3%**	$1,019	$513	**50.3%**
GPD	0.925	50,000	$643	$1,079	**$436**	**67.8%**	$645	$1	**0.2%**	$1,535	$792	**51.6%**
GPD	0.99	27,500	$636	$1,121	**$486**	**76.4%**	$637	$2	**0.3%**	$1,698	$828	**48.8%**
TGPD	0.775	33,500	$141	$214	**$73**	**52.0%**	$144	$3	**2.2%**	$297	$170	**57.4%**
TGPD	0.8	25,000	$140	$220	**$80**	**56.9%**	$145	$5	**3.3%**	$315	$179	**56.8%**
TGPD	0.8675	50,000	$452	$737	**$285**	**63.0%**	$466	$13	**3.0%**	$1,062	$576	**54.3%**
TGPD	0.91	31,000	$451	$761	**$309**	**68.6%**	$463	$12	**2.7%**	$1,174	$603	**51.4%**
TGPD	0.92	47,500	$698	$1,149	**$451**	**64.7%**	$704	$7	**0.9%**	$1,668	$888	**53.2%**
TGPD	0.95	35,000	$717	$1,206	**$489**	**68.2%**	$715	-$2	**-0.2%**	$2,009	$991	**49.3%**

*NOTE: #simulations = 1,000; λ = 25 for 10 years so n ~ 250; α=0.999

TABLE 4b: RCE vs. MLE ECap Distributions – Bias and RMSE by Severity by Parameter Values ($millions)

Severity Dist.	Parm1 μ	Parm2 σ	True ECap	Mean MLE ECap	MLE Bias	MLE Bias%	Mean RCE ECap	RCE Bias	RCE Bias%	RMSE MLE ECap	RMSE RCE ECap	RMSE ECap RCE/MLE
LogN	10	2	$107	$115	**$8**	**7.8%**	$108	$1	**1.1%**	$47	$43	**91.3%**
LogN	7.7	2.55	$107	$121	**$14**	**13.2%**	$109	$3	**2.5%**	$66	$57	**87.0%**
LogN	10.4	2.5	$1,286	$1,449	**$163**	**12.7%**	$1,316	$30	**2.4%**	$769	$673	**87.4%**
LogN	9.27	2.77	$1,293	$1,498	**$205**	**15.8%**	$1,333	$40	**3.1%**	$898	$764	**85.2%**
LogN	10.75	2.7	$4,230	$4,864	**$634**	**15.0%**	$4,352	$123	**2.9%**	$2,828	$2,425	**85.8%**
LogN	9.63	2.97	$4,303	$5,097	**$794**	**18.5%**	$4,461	$158	**3.7%**	$3,321	$2,769	**83.4%**
TLogN	10.2	1.95	$126	$144	**$18**	**14.7%**	$124	-$2	**-1.3%**	$101	$77	**76.1%**
TLogN	9	2.2	$133	$179	**$46**	**35.0%**	$131	-$2	**-1.5%**	$202	$97	**48.0%**
TLogN	10.7	2.385	$1,267	$1,678	**$411**	**32.4%**	$1,338	$71	**5.6%**	$1,521	$1,003	**65.9%**
TLogN	9.4	2.65	$1,297	$1,966	**$669**	**51.6%**	$1,264	-$33	**-2.5%**	$2,910	$1,192	**41.0%**
TLogN	11	2.6	$4,208	$5,639	**$1,431**	**34.0%**	$4,337	$129	**3.1%**	$6,319	$4,072	**64.4%**
TLogN	10	2.8	$4,145	$6,279	**$2,134**	**51.5%**	$4,177	$32	**0.8%**	$8,119	$3,972	**48.9%**
	a	**b**										
Logg	24	2.65	$192	$225	**$33**	**17.0%**	$199	$7	**3.4%**	$163	$137	**84.1%**
Logg	33	3.3	$203	$225	**$22**	**10.7%**	$204	$1	**0.4%**	$131	$115	**87.5%**
Logg	25	2.5	$1,064	$1,272	**$208**	**19.5%**	$1,105	$42	**3.9%**	$984	$814	**82.7%**
Logg	34.5	3.15	$960	$1,090	**$130**	**13.5%**	$978	$18	**1.8%**	$734	$631	**86.0%**
Logg	25.25	2.45	$1,877	$2,300	**$423**	**22.5%**	$1,986	$109	**5.8%**	$1,842	$1,507	**81.8%**
Logg	34.7	3.07	$1,794	$2,097	**$302**	**16.8%**	$1,869	$74	**4.1%**	$1,500	$1,273	**84.9%**
TLogg	23.5	2.65	$271	$496	**$225**	**83.1%**	$297	$27	**9.9%**	$903	$530	**58.6%**
TLogg	33	3.3	$261	$382	**$120**	**46.1%**	$247	-$15	**-5.6%**	$458	$191	**41.6%**
TLogg	24.5	2.5	$1,164	$2,152	**$988**	**84.9%**	$1,099	-$65	**-5.6%**	$3,620	$1,370	**37.8%**
TLogg	34.5	3.15	$1,086	$1,437	**$350**	**32.2%**	$1,158	$72	**6.6%**	$1,453	$941	**64.8%**
TLogg	24.75	2.45	$1,928	$3,618	**$1,689**	**87.6%**	$1,855	-$74	**-3.8%**	$6,604	$2,269	**34.4%**
TLogg	34.6	3.07	$1,892	$2,493	**$601**	**31.8%**	$2,050	$158	**8.4%**	$2,499	$1,764	**70.6%**
	ξ	**θ**										
GPD	0.8	35,000	$382	$696	**$313**	**81.9%**	$396	$13	**3.5%**	$1,069	$521	**48.7%**
GPD	0.95	7,500	$375	$785	**$410**	**109.2%**	$390	$14	**3.8%**	$1,398	$588	**42.1%**
GPD	0.875	47,500	$1,106	$2,123	**$1,016**	**91.9%**	$1,130	$24	**2.2%**	$3,514	$1,594	**45.4%**
GPD	0.95	25,000	$1,251	$2,576	**$1,325**	**105.9%**	$1,279	$28	**2.2%**	$4,585	$1,930	**42.1%**
GPD	0.925	50,000	$1,938	$3,835	**$1,898**	**97.9%**	$1,955	$17	**0.9%**	$6,657	$2,882	**43.3%**
GPD	0.99	27,500	$2,076	$4,375	**$2,299**	**110.7%**	$2,095	$19	**0.9%**	$8,085	$3,275	**40.5%**
TGPD	0.775	33,500	$351	$617	**$266**	**75.7%**	$365	$13	**3.8%**	$1,109	$543	**48.9%**
TGPD	0.8	25,000	$361	$660	**$299**	**82.9%**	$379	$18	**5.0%**	$1,193	$579	**48.5%**
TGPD	0.8675	50,000	$1,267	$2,432	**$1,166**	**92.0%**	$1,327	$61	**4.8%**	$4,337	$1,988	**45.9%**
TGPD	0.91	31,000	$1,334	$2,672	**$1,338**	**100.4%**	$1,389	$55	**4.1%**	$5,165	$2,203	**42.6%**
TGPD	0.92	47,500	$2,088	$4,048	**$1,960**	**93.9%**	$2,129	$41	**2.0%**	$7,203	$3,235	**44.9%**
TGPD	0.95	35,000	$2,227	$4,474	**$2,246**	**100.8%**	$2,246	$19	**0.8%**	$9,606	$3,873	**40.3%**

*NOTE: #simulations = 1,000; λ = 25 for 10 years so n ~ 250; α=0.9997

sometimes a larger value for "a" induces more bias at the same level of capital, and sometimes larger (smaller) values of "b" induce more bias at the same level of capital.

The bias of MLE-based capital estimates is material for, arguably, every result generated by this simulation study at $\lambda = 25$. Even for the lowest capital bias in absolute terms in Table 4a – $4m corresponding to a true capital value of $63m under a (non-truncated) LogNormal severity – it is difficult to argue that pushing the button to implement RCE in under a second is not justified. And capital typically is subject to far greater inflation – sometimes even more than double true capital in relative terms, or well over $2b beyond true capital in absolute terms (see Table 4b). The larger numbers generally are more relevant to the larger banks, mainly because the parameter values of the severities typically are larger for them. But even for the smaller and medium-sized banks, it is important to remember that these numbers are *per UoM*. The cumulative effect of bias from every UoM is likely to be quite large, even for those banks not classified as the largest, and even after diversification benefits are taken into account. In contrast to MLE-based capital estimates, the accuracy of RCE is always within ±11% of true capital, except for when $\lambda = 15$ where 11 of 72 RCE simulations (about 15%) deviated more than 11% from true capital (but were still much closer to true capital – see Tables F5a,b).

TABLE 5: Summary of Capital Accuracy by Sample Size – MLE vs. RCE ($millions)

	+------------------- ECap -------------------+				+------------------- RCap -------------------+			
	Mean of Absolute Bias		Median of Absolute Bias		Mean of Absolute Bias		Median of Absolute Bias	
λ =	**RCE**	**MLE**	**RCE**	**MLE**	**RCE**	**MLE**	**RCE**	**MLE**
15	7.8%	92.6%	2.6%	82.3%	5.9%	61.6%	1.6%	58.1%
25	3.4%	53.1%	3.3%	40.6%	2.4%	38.1%	2.0%	30.6%
50	2.8%	25.7%	2.7%	17.7%	2.0%	19.4%	1.9%	14.3%
75	1.2%	15.5%	0.8%	10.7%	0.8%	11.9%	0.5%	8.7%
100	0.9%	11.3%	0.5%	7.9%	0.5%	8.7%	0.4%	6.1%
15	$61	$825	$18	$502	$21	$228	$5	$154
25	$45	$727	$29	$410	$14	$209	$8	$133
50	$69	$617	$52	$320	$20	$182	$15	$109
75	$40	$526	$14	$250	$11	$157	$3	$80
100	$32	$485	$15	$223	$7	$142	$5	$73

A summary of capital accuracy from Tables F4a,b-F8a,b, in both relative and absolute terms, for MLE and RCE across sample sizes is shown in Table 5. Obviously this is heavily dependent on the severities used in this study (although they are the most widely used in practice), but it demonstrates how, even when MLE bias shrinks in relative terms as sample sizes increase, in absolute terms it remains quite large. RCE bias, in contrast, always is far smaller. It also is important to note that on average, RCE was slightly larger than true capital. This is important from the perspective of conservatism: any estimator should avoid even the appearance of benefiting from use merely to decrease required capital below what is consistent with regulatory intent (which, here, is estimation closest to the true capital numbers).

In sum, the inflation bias of MLE-based capital estimates can be enormous under conditions that are not uncommon, and that of RCE typically is small, and often de minimis: except for a few cases of smaller sample sizes (when $\lambda = 15$), RCE essentially and effectively eliminates capital bias due to Jensen's inequality.

Capital Precision and Model Stability

Model stability long has been cited by many as the most important and most difficult challenge for AMA operational risk capital estimation. Some of the major factors contributing to model (in)stability in this setting include the quality of the samples of loss event data (or lack thereof), notable data paucity, the arguably inherent heterogeneity of operational risk UoM definition, the (non-)robustness of the parameter estimators selected and used, low statistical power in the choice of severity, and critically, the size of the variance of the output of the model (here, the capital estimates). Achieving adequate precision in the model output under ideal, textbook data conditions unarguably is the first step in achieving broader model stability. Put differently, if under idealized conditions (e.g. perfectly homogenous i.i.d. loss event data) a model framework cannot generate capital estimates that are precise enough to use to make reliable inferences about the true values of capital, then there is no way that improving the other components of model stability will ever make a model "stable." The starting point for achieving and/or testing for model stability is testing the model when the data satisfies all, or nearly all of the assumptions required of the model (e.g. i.i.d. data). If a model fails this test, it will never achieve "model stability," and so this is the capital precision test applied here to RCE and standalone MLE.

Capital precision is measured by a number of descriptive statistics of the MLE-based and RCE-based capital distributions, including the standard deviation, the coefficient of variation, the RMSE (which also incorporates bias), the inter-quartile range, and empirical 95% confidence intervals. Although not statistics of "spread," relevant, too, are the skewness and kurtosis of these distributions. By every single one of these criteria, in every single simulation, RCE is more precise than MLE, and sometimes dramatically so (it also is less skewed and less kurtotic than MLE in all cases). This is a very strong result, although not unexpected because RCE capital is essentially scaled MLE capital, where the scaling factor, which is always less than one and greater than zero, is a function of the degree of apparent convexity of VaR for a given sample size and severity: the more convex is VaR for a particular estimate, the smaller the value of the scaling factor. Most notably, RMSE of RCE-based capital estimates is less than half that of MLE-based capital estimates in fully half of the ECap basecase simulations (see Table 4b). The numbers are similar for the standard deviations of the capital distributions of

RCE vs. MLE, and even the inter-quartile range, which is less affected by MLE's extreme outliers than is RMSE or standard deviation, sometimes is less than half the value for RCE compared to MLE. Importantly, the empirical 95% confidence intervals, which are embarrassingly large for MLE, are much smaller for RCE, on average only two thirds the size across both the RCap and ECap results; and in one eighth of all cases, they are less than half the size of those of MLE. By any measure, RCE-based capital estimates are notably more precise than MLE-based capital estimates.

Capital Robustness

Tables F9a,b-F11a,b in Appendix F show the equivalent of the basecase Tables 4a,b for $\lambda = 25$ but with 5% right-tail contamination, 5% left-tail contamination, and 10% both-tail contamination, respectively, where contamination[67] comes from the same distribution with parameter values at either end of the 90% confidence interval of the joint parameter distribution, as described above. For all right-tail contamination simulations,[68] RCE capital still is much less biased than is MLE capital in absolute terms, but this is not what matters when assessing robustness since the capital distribution is now "wrong" – it no longer represents the original "true" capital numbers because it is contaminated. What matters is how much capital deviates from their original estimates under no contamination, i.e. under i.i.d. data samples. The average absolute deviation of MLE economic capital is 18.9% – more than twice as large as that of RCE economic capital (8.7%). Because ECap is subject to greater convexity farther out in the right tail, these numbers are larger than the respective RCap numbers – 11.8% and 6.4% – but this relative difference is still notable. That these differences are not even larger arguably is mostly a function of RCE's utilization of MLE as its capital estimator. While this was necessary for an apples-to-apples comparison to MLE that isolates the effects of using RCE, if RCE was used with a more robust severity estimator, these differences likely would be larger, ceteris paribus.

For all left-tail contamination simulations, RCE capital again is much less biased than MLE capital in absolute terms, but what matters is the extent to which capital deviates from the original estimates under no contamination, i.e. under i.i.d. data samples. For ECap, MLE deviates an average of 9.5% while RCE deviates an average of 6.5%. The respective numbers for RCap were 7.2% and 5.2%. Because we are dealing with the estimation of extremely large quantiles, it is not surprising that contamination in the other direction has less effect on the extent to which either estimator deviated from its i.i.d. values. But again, RCE deviated less.

[67] Note again that the %contamination is stochastic.

[68] Note that two Truncated LogGamma severities – those with the smallest values of b – required larger percentages of right-tail contamination to achieve some stability in their estimation. This is not surprising given that this parameter drives the extreme tail of the distribution, and estimation of such a heavy-tailed distribution is difficult even under i.i.d. conditions when data samples are not large; so under contaminated conditions, estimation difficulties are not uncommon. This was also true for one of the Truncated LogGamma distributions and one of the GPD distributions under left-tail contamination.

Finally, under both-tail contamination, for ECap MLE deviated an average of 6.3% while RCE deviated an average of 3.1%. The respective numbers for RCap were 4.1% and 2.1%.

In sum, while the increased robustness of RCE capital over MLE capital is consistent, is it not as notable as the strong capital precision gains provided by RCE or the dramatic capital accuracy gains provided by RCE over MLE. This is at least in part due to the reliance on MLE estimates in this study before the RCE capital adjustment is applied. Also, the largest robustness advantage of RCE over MLE occurred where robustness arguably matters most: under non-i.i.d. data in the extreme right tail of the aggregate loss distribution. Finally, it is worth noting that the deviations violated the expected order of the contaminated simulations more often for MLE capital than for RCE capital. In other words, for a given severity, in general one would expect a larger negative deviation under left-tail contamination, a smaller negative or a smaller positive deviation under both-tail contamination, and a large positive deviation under right-tail contamination. But due to random sampling error these comparisons "crossed over" each other, violating this order 9 of the 72 possible comparisons for MLE capital.[69] Given the conditions of the estimation exercise this is not surprising. But it is noteworthy that, in contrast, this happened only 3 times for RCE capital.[70] This is consistent with a more stable, less variable, and more robust capital estimator.

Discussion

"If you can't measure something, you can't understand it. If you can't understand it, you can't control it. If you can't control it, you can't improve it." - H. J. Harrington

I restate Harrington's measurement dictum here because operational risk capital modeling is all about measurement, yet it faces serious constraints and obstacles that span the empirical, the methodological, and the regulatory that make its measurement extremely challenging. As examined in this paper, these constraints often interact in material and complex ways: estimating an exceedingly large severity quantile (regulatory and methodological constraint) of medium- to heavy-tailed severities (empirical and regulatory constraint) under sample sizes that typically are fairly small and rarely "large" (empirical constraint) and subject to biasing effects apparently due to Jensen's inequality (methodological constraint) all converge in a perfect storm of estimation challenges that make what appears to be a fairly straightforward framework (LDA) arguably virtually unusable, as currently implemented across the industry, for operational risk mitigation purposes because it cannot

[69] No results showed a "double" violation, that is, none showed estimated capital from a left-tail deviation that was larger than that from a right-tail deviation.

[70] Under a null hypothesis of 9 "cross-overs" out of 72 comparisons ($p = 0.125$) and a binomial sample space, the probability of 3 or fewer cross-overs occurring is only about p = 0.016.

effectively measure and estimate capital. One arrives at this conclusion not only based on the results provided in this paper, but also according to some of the most respected and established operational risk practitioners (see OR&R, 2013). Absent any one of the factors listed above, the deleterious effects on capital estimation either are notably mitigated or disappear altogether, but unfortunately their simultaneous occurrence in any given UoM is not uncommon, and arguably common.

RCE was designed specifically to address these issues head-on. As shown above, RCE-based capital is dramatically more accurate, notably more precise, and modestly though consistently more robust than is MLE-based capital. Because RCE uniformly lowers capital requirements at the unit-of-measure level, it must do the same at the enterprise level, and likewise must increase capital stability from quarter to quarter, ceteris paribus. This decrease in capital, however, is not merely consistent with regulatory intent, but rather, is arguably *more* consistent with regulatory intent than most, if not all other implementations of LDA. We can say this simply because other implementations generate capital estimates that are systematically and materially biased upwards under conditions that are not uncommon. Regulatory intent cannot possibly support systemically and materially biased capital estimation but rather, unbiased capital estimation: in other words, capital estimates that, when based on multiple samples of loss event data, form a distribution with an *expected value* that is centered on true capital.[71] That regulators also would expect this unbiased capital estimator to be reasonably precise and robust to make reliable inferences about the true values of capital merely is consistent with their stated requirements for "credible … and verifiable processes … that most effectively enables it [the regulated bank/sifi] to quantify its exposure to operational risk" (see US Final Rule, 2007).

[71] One question raised about the capital distributions of LDA-based MLE vs. RCE is whether the median of the former is closer to true capital than is that of the latter, even though the mean of the latter is dramatically closer to true capital (which is to say RCE is essentially unbiased, whereas MLE often is very biased). An empirical examination of all the (i.i.d.) capital distributions generated in this study (360) shows this to be true: LDA-based MLE is consistently more "median-centered" than is RCE. On its face this might appear to be a comparative advantage of LDA-based MLE, but one must ask, "At what cost?" By every single measure of spread, RCE is consistently, if not dramatically more precise than is MLE: always. RCE also is systematically less skewed and less kurtotic than is MLE. So the cost of MLE's "median-centeredness" is an extremely long capital tail, which is exactly the thing all operational risk managers and analysts are trying to avoid the most. Also, we must question whether we are asking the right question: what is important is not whether a specific quantile (e.g. the median) of a distribution is closer to the true value of the estimate (here, capital), but rather, whether the entire distribution of a statistic is closer to the true value compared to that of another? RMSE is one metric, arguably the best one, that answers this question, and the RMSE of RCE is always notably, if not dramatically better (smaller) than that of MLE. Another way to answer address this issue is to find the cross-over points of the 72 baseline (i.e. $\lambda = 25$) capital distributions: that is, find the percentile at which the right tail of RCE-based capital becomes closer to true capital than does that of MLE-based capital. A percentile close to the median would indicate a very narrow range over which MLE arguably has any advantage. And the empirical answer is that all cross-over points are below the 62%tile, and over two thirds are below the 60%tile. So for the percentiles that matter (i.e. the right tail), RCE capital always is closer to true capital than is MLE capital.

Conclusions

As we have seen above, RCE satisfies all the preferred criteria governing its development listed at the beginning of this paper. The typical consequences for large banks of using RCE rather than standalone MLE in their LDA frameworks will be: i) in many cases, notable reductions in capital, both for RCap and ECap, at both the UoM and enterprise levels; ii) in cases where capital inflation presumably due to Jensen's inequality is material, capital dramatically closer to "true" capital as defined by the LDA framework (and thus, capital estimates more consistent with and closer to that intended by regulators); iii) greater precision in these capital estimates, so capital will vary less and remain more stable from quarter to quarter, all else equal; and iv) more robustness to violations of the i.i.d. data assumption that plague even the most well-defined UoM's.

Smaller and medium-sized banks, whose severity parameter values typically will be smaller, all else equal, still should enjoy notable reductions in capital when RCE reduces bias across all UoM's, and they should enjoy the precision and robustness benefits of RCE as well. And to reemphasize yet again, all these reductions are *more* consistent with regulatory intent than are the MLE-based capital estimates under an LDA framework, so RCE unambiguously provides the proverbial "win-win." The capital estimator that systematically reduces the operational risk capital that banks and SIFIs must hold in reserve also is the one that gets to the "right" regulatory capital number. So there seems to be little to argue against the widespread use of RCE for AMA(LDA)-based operational risk capital estimation.

Areas of future research related to RCE include: i) Even though RCE dramatically mitigates capital inflation apparently due to Jensen's inequality, more needs to be done on the capital precision and capital robustness fronts. Where MLE-based capital variability is largest, RCE decreases variability the most: for example, for a GPD severity with $\xi = 0.99$ and $\theta = 27{,}500$, as shown on Table 4b, RMSE of MLE-based capital is about \$8.1b, but RMSE of RCE-based capital is only 39% of this value at \$3.3b. This is a large relative decrease, but in absolute terms \$3.3b is still a very large number (especially when true capital is \$2.1b!), and decreasing it further is vitally important for the capital planning that banks and SIFIs subject to these regulations must do. This may be one area, however, where methodological innovation may be limited by what many applied statisticians would call an ill-posed problem: that is, expecting to estimate severity quantiles associated with $p = 0.99999$ and higher with anything approaching a reasonable degree of precision. For example, Schevchenko (2011) uses a very straightforward calculation to show that under reasonable assumptions, even under i.i.d. loss data, sample size requirements for obtaining reasonable precision in the capital estimate are on the order of magnitude of a million or more loss events, or 50,000 to 100,000 years worth of loss data. Another way of stating this is that the variance associated with estimating extremely large quantiles – even taking any effects of

Jensen's inequality out of the equation – inherently is extremely large, especially for the heavy-tailed severities that must be used in this setting. It may therefore take radical innovation and dramatic advances in applied statistics to even partially circumvent such a daunting challenge.

ii) A general mathematical proof of VaR's multidimensional surface convexity for extremely large quantile estimation for all the severities relevant to operational risk capital estimation would be desirable. However, it arguably is not strictly necessary in this setting since the number of severity distributions "allowed" and used in practice is quite finite, and the VaR of each at least can be checked for convexity in two straightforward ways: a) graphical checks for marginal convexity, as is done in Appendix A, Figure A1, and b) via a comparison of the mean of very straightforward capital simulations to true capital, which is required anyway to determine whether the size of the parameter estimates makes the capital bias material. Still, additional mathematical confirmation of these empirical findings would further validate them and more specifically define the conditions under which they hold.

iii) An analytic derivation of the values of $c(sev, n)$ as a function not only the severity distribution and the sample size, but also of the severity parameter values and the size of the quantile being estimated could be useful. If possible, this likely would improve RCE capital estimates under those few smaller-sample conditions where its bias is not negligible (see Tables F5a,b). And in Tables F6a,b, F7a,b, and F8a,b, we can see some differences in RCE accuracy by the size of the quantile being calculated, that is, by whether we look at RCap or ECap. Although RCE still vastly outperforms MLE in all these specific cases, it possibly could be improved if its value was based on an analytical derivation of its relationship with parameter values and quantile size.

Additionally, while a derivation of $c(sev, n)$ as an analytic function of sample size would be preferable to the empirical approach used in this paper, this is arguably more desirable than it is necessary because the range covered here spans most sample sizes where bias is material. This is true, also, of a derivation that holds across severity distributions because of the conditional nature of capital estimation under LDA: requiring knowledge of $c(sev, n)$ *after* the severity has been selected does not invalidate the ex ante nature of the capital estimation.

In the end, analytic derivations supporting empirical results always are preferable to empirical results alone, so such derivations would at the very least further validate the findings of this study and continue to rightly encourage focus of the research on operational risk capital estimation *on the capital distribution*, where it belongs. So at least for these purposes, such derivations are worth pursuing.

iv) As mentioned above, testing RCE's robustness based on violations of the independence assumption and not just the identically distributed assumption would be useful, especially since operational risk loss event data is likely to at least sometimes be serially correlated. Testing EVT-POT, spliced distributions, and kernel transformations for capital bias due to Jensen's inequality also could be important follow-ons to this paper.

I conclude by tying this research back to the broader operational risk setting with a recent quote from the head of a major US regulatory agency: "… it [operational risk] is currently at the top of the list of safety and soundness issues for the institutions we supervise. This is an extraordinary thing. Some of our most seasoned supervisors, people with 30 or more years of experience in some cases, tell me that this is the first time they have seen operational risk eclipse credit risk as a safety and soundness challenge. Rising operational risk concerns them, it concerns me, and it should concern you…" (Thomas J. Curry, Comptroller of the Currency, OCC, Before the Exchequer Club, May 16, 2012). From the perspective of operational risk capital estimation, the ominous tone here should trigger alarm based on the fact that the most widely used methods of estimating operational risk capital (such as MLE) under the most widely used framework (LDA) generate capital estimates that often are i) demonstrably, materially, and systematically inflated, and thus, utterly inconsistent with regulatory intent; ii) grossly imprecise by any measure; and iii) based on relatively fragile, mathematically convenient, idealized textbook assumptions that are consistently violated by real world data. This paper is the first to directly and comprehensively address these three issues by not only quantifying them, but also by i) identifying one of their major analytical sources, ii) specifying exactly the conditions under which they are material, and then iii) actually designing an approach to address, if not solve them. The broader objective, however, is to spur related research on these topics and focus more attention on the capital distribution. After all, capital estimation, not parameter estimation, is the endgame here, and as much time and resources must be dedicated to this as have been dedicated to research on severity parameter estimation if we are to make the existing framework more useable and useful in practice, not to mention more consistent with regulatory intent. Without this focus, the empirical evidence presented herein regarding MLE-based capital estimation under LDA transforms H. J. Harrington's aphorism on measurement and improvement into a dire warning that echoes Curry's concern. Simply put, we cannot improve business operations by effectively mitigating operational risk if the capital we estimate to represent it is not measured with reasonable accuracy, reasonable precision, or reasonable robustness.

In part because this paper began on a critical note, I would like to finish on a thought-provoking methodological note of optimism by placing RCE, and any similar capital-distribution-based research efforts, in a forward-looking context. The apparent effects of Jensen's inequality often are very damaging to capital estimation in this setting, and the approach taken by RCE is to control and eliminate them. But what if we could go a step further and actually exploit them? By this I mean, what if we could develop an estimator that not only mitigates or eliminates the biasing, imprecision, and non-robustness effects of convexity, but also becomes *less* rather than more biased, and *more* rather than less precise, and *more* rather than less robust, in the face of convexity? What if we could develop an estimator whose statistical properties actually *improve* under conditions of convexity? It is very telling that until the recent publication of Nicolas Nassim Taleb's book "Antifragile: Things that Gain from Disorder" (Taleb, 2014), there was not even a word to describe an estimator with such "antifragile" characteristics (note that "antifragility" is very distinct from "robustness").[72] But such an "antifragile" estimator would be enormously useful in this and many other risk settings where convexity is common, if not endemic, and plagues the effective use of a number of the most commonly used risk metrics (expected shortfall included), whether or not their users are aware of it (the need for this paper is a case in point). So regarding convexity and its deleterious effects on estimation generally, for capital and otherwise, we should aim high: first we must test for it, identify it, and measure it; then we must develop estimators like RCE to control it and in many ways, eliminate it; and finally, an ultimate goal would be to exploit it using "antifragile" estimators that actually *improve* when confronting it.

References

Ames, M., Schuermann, T., and Scott, H., (2014), "Bank Capital for Operational Risk: A Tale of Fragility and Instability," Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2396046.

Alemany, R., Bolancé, C., Guillén, M., (2012), "Nonparametric Estimation of Value-at-Risk," Working Paper.

Andreev, V., Tinyakov, S., Parahin, G., and Ovchinnikova, O., (2009), "An Application of EVT, GPD, and POT Methods in the Russian Stock Market," Working Paper, http://ssrn.com/abstract=1507678.

Artzner, P., Delbaen, F., Eber, J., and Heath, D., (1999), "Coherent Measures of Risk," *Mathematical Finance*, **9**, 203-228.

Arakelian, V., and Karlis, D., (2014), "Clustering Dependencies via Mixtures of Copulas," *Communications in Statistics – Simulation and Computation*, Vol.43(7).

Basel Committee on Banking Supervision, (2004), "International Convergence of Capital Measurement and Capital Standards: A Revised Framework," Bank for International Settlements, June, 2004.

Basel Committee on Banking Supervision, (2011), "Operational Risk – Supervisory Guidelines for the Advanced Measurement Approaches," Bank of International Settlements. Available online at http://www.bis.org/publ/bcbs196.htm

Bernard, C. and Vanduffel, S., (2014), "A New Approach to Assessing Model Risk in High Dimensions," Working Paper, http://ssrn.com/abstract=2393054

Böcker, K., and Klüppelberg, C. (2005), "Operational VaR: A Closed-Form Approximation," RISK Magazine, 12, 90-93.

Böcker, K., and Sprittulla, J. (2006), "Operational VaR: meaningful means." RISK Magazine, 19 (12), 96–98.

[72] For a mathematical treatment of antifragility, see Taleb and Douady (2013), and for an application to bank stress tests, see Taleb et al. (2012). The fragility heuristic (H) in both papers actually is conceptually similar to RCE in that both are measures of convexity based on perturbations of parameters: H measures the distance between the average of model results over a range of shocks and the model result of the average shock, while RCE is a scaling factor based on the ratio of the median to the mean of similar parameter perturbations. Both exploit Jensen's inequality to measure convexity: in the case of the fragility heuristic, to raise an alarm about it, and in the case of RCE, to eliminate it (or rather, to effectively mitigate its biasing effects on capital estimation).

Bolancé, C., Guillén, M., Gustafsson, J., and Nielsen, J., (2012), Quantitative Operational Risk Models, ed. M. Ong, CRC Press, Taylor & Francis Group, Boca Raton, FL.

Bolancé, C., Guillén, M., Gustafsson, J., and Nielsen, J., (2013), "Adding Prior Knowledge to Quantitative Operational Risk Models," The Journal of Operational Risk, Vol. 8(1), 17-32.

Box, G., (1984), "The Importance of Practice in the Development of Statistics," *Technometrics*, 26(1), 1-8.

Box, G., and Luceño, A., (1998), "Quality Quandaries: Models, Assumptions, and Robustness," *Quality Engineering*, 10(3), 595-598.

Boyd, S. and Vandenberghe, L., (2009), Convex Optimization, Cambridge University Press, Cambridge, UK.

Cavallo, A., Rosenthal, B., Wang, X., and Yan, J., (2012), "Treatment of the Data Collection Threshold in Operational Risk: A Case Study Using the Lognormal Distribution," *The Journal of Operational Risk*, 7(1), 3-38.

Cope, E., Mignola, G., Antonini, G., and Ugoccioni, R. (2009), "Challenges and Pitfalls in Measuring Operational Risk from Loss Data," *The Journal of Operational Risk*, 4 (4), 3–27.

Cope, E. (2011), "Penalized likelihood estimators for truncated data," *Journal of Statistical Planning and Inference*, 141 (1), 345-358.

Curry, T., (2012), "Before the Exchequer Club," Speech delivered May 16, 2012.

Danielsson, J., Jorgensen, B., Samorodnitsky, G., Sarma, M., and de Vries, C. (2005), "Subadditivity Re-Examined: the Case for Value-at-Risk," Working Paper.

Degen, M., and Embrechts, P., (2011), "Scaling of High-Quantile Estimators," *Journal of Applied Probability*, 48(4), 968-983.

Degen, M., Embrechts, P., and Lambrigger, D., (2007), "The Quantitative Modeling of Operational Risk: Between g-and-h and EVT," *Astin Bulletin*, 37(2), 265-291.

Degen, M. (2010), "The Calculation of Minimum Regulatory Capital Using Single-Loss Approximations," *The Journal of Operational Risk*, 5(4), 3-17.

Dhaene, J., Linders, D., Schoutens, W., and Vyncke, D., (2013), "A Multivariate Dependence Measrue fo Aggregating Risks," Working Paper, http://ssrn.com/abstract=2380707

Duan, N. (1983), "Smearing Estimate: A Nonparametric Retransformation Method," Journal of the American Statistical Association, 78, 605-610.

Dutta, P., and Perry, J., (2006), "A Tale of Tails: An Empirical Analysis of Loss Distribution Models for Estimating Operational Risk Capital," Working Paper No. 06-13, Federal Reserve Bank of Boston, MA.

Nešlehová, J., Embrechts, P., and Chavez-Demoulin, V., (2006), "Infinite-mean Models and the LDA for Operational Risk," *The Journal of Operational Risk*, 1(1), 3-25.

Embrechts, P. and Neslehova, J., (2006), "Quantitative Models for Operational Risk," Presentation.

Embrechts, P., and Frei, M., (2009), "Panjer Recursion versus FFT for Compound Distributions," Mathematical Methods of Operations Research, Vol. 69(3), 497-508.

Ergashev, B. (2008), "Should Risk Managers Rely on the Maximum Likelihood Estimation Method while Quantifying Operational Risk?" *The Journal of Operational Risk*, 3 (2), 63-86.

Ergashev, B., (2009), "Estimating the Lognormal-Gamma Model of Operational Risk Using the Markov Chain Monte Carlo Method," *The Journal of Operational Risk*, Vol. 4(1), 35-57.

Ergashev, B., Pavlikov, K., Uryasev, S., and Sekeris, E., (2014), "Estimation of Truncated Data Samples in Operational Risk Modeling," Working Paper.

European Commission, MEMO/07/286, July, 2007, "'SOLVENCY II': Frequently Asked Questions (FAQs)".

Federal Register, Rules and Regulations, (2007) Vol. 72, No. 235, December 7.

Finkelstein, M., Tucker, H., and Veeh, J., (2006), "Pareto Tail Index Estimation Revisited," *North American Actuarial Journal*, Vol. 10(1).

Fisher, L., Weaver, D., and Webb, G. (2009), "Removing Bias in Computed Returns," working paper, Electronic copy available at: http://ssrn.com/abstract=1420314.

Fölmer, H., and Schied, A., (2002), "Convex Measures of Risk and Trading Constraints," *Finance and Stochastics*, **6**, 429-447.

Frittelli, M., and Gianin, E., (2002), “Putting Order in Risk Measures,” *Journal of Banking and Finance*, **26**, 1473-1486.

Föllmer, H, and Schied, A. (2011), Stochastic Finance–An Introduction in Discrete Time, Graduate Textbook Series. 3rd edition, Walter de Gruyter & Co., Berlin.

Guégan, D., and Hassani, B., (2013), “Using a Time Series Approach to Correct Serial Correlation in Operational Risk Capital Calculation,” *The Journal of Operational Risk*, 8(3), 31-56.

Gustafsson, J., and Nielsen, J., (2008), “A Mixing Model for Operational Risk,” *The Journal of Operational Risk*, 3(3), 25-37.

Hampel, F.R., E. Ronchetti, P. Rousseeuw, and W. Stahel. (1986), Robust Statistics: The Approach Based on Influence Functions, John Wiley and Sons, New York.

Haubenstock, M., and Hardin, L., (2003), “The Loss Distribution Approach,” Chapter 8 in Operational Risk: Regulation, Analysis, and Management, C. Alexander, Ed., Financial Times Prentice Hall, New York.

Hernandez, L., Tejero, J., Suarez, Al., and Carrillo-Menendez, S., (2013), “Closed-form Approximations for Operational Value-at-Risk,” *The Journal of Operational Risk*, 8(4), 39-54.

Hess, C., (2011), “Can the Single-Loss Approximation Method Compete with the Standard Monte Carlo Simulation Technique?,” *The Journal of Operational Risk*, 6(2), 31–43.

Horbenko, N., Ruckdeschel, P. and Bae, T. (2011), “Robust Estimation of Operational Risk,” *The Journal of Operational Risk*, 6 (2), 3-30.

Huber, P. and Ronchetti, E., (2009), Robust Statistics, Second Edition, John Wiley & Sons, New York.

Hull, J., (2012), Risk Management and Financial Institutions, 3rd Ed., John Wiley & Sons, Inc., Hoboken, New Jersey.

Hyung, N., and de Vries, C., (2007), “Portfolio Selection with Heavy Tails,” *Journal of Empirical Finance*, Vol.14(3), 383-400.

Ibragimov, R., (2008), “Portfolio Diversification and Value at Risk Under Thick-Tailedness,” Harvard University Working Paper.

Interagency Guidance, (2011), “Interagency Guidance on the Advanced Measurement Approaches for Operational Risk,” Board of Governors of the Federal Reserve System, Federal Deposit Insurance Corporation, Office of the Comptroller of the Currency, Office of Thrift Supervision, June 3, 2011.

Jensen, J. L. W. V. (1906). "Sur les fonctions convexes et les inégalités entre les valeurs moyennes," *Acta Mathematica*, 30 (1): 175–193.

Jiao, X. and Zha, H. (2008), "Consistent Computation of First- and Second-Order Differential Quantities for Surface Meshes," SPM '08 Proceedings of the 2008 ACM Symposium on Solid and Physical Modeling, pp.159-170.

Johnson, R., and Wichern, D., (2007), Applied Multivariate Statistical Analysis, 6th Ed., Pearson, Upper Saddle River, NJ.

Jorian, P., (2000), Value at Risk: The New Benchmark for Managing Financial Risk, 2nd Ed., McGraw-Hill, New York, New York.

Joris, C., (2013), “Robust Estimation in Operational Risk Modeling,” Masters Thesis.

Kato, T., (2013), “Quantitative Operational Risk Management: Properties of Operational Value at Risk (OpVaR),” Working Paper, Osaka University.

Kennedy, P. (1992), A Guide to Econometrics, The MIT Press, Cambridge, Massachusetts.

Kim, J., (2010), “Conditional Tail Moments of the Exponential Family and its Related Distributions,” *North American Actuarial Journal*, Vol. 14(2), 198-216.

Kim, J., and Hardy, M., (2007), “Quantifying and Correcting the Bias in Estimated Risk Measures,” *Astin Bulletin*, Vol. 37(2), 365-386.

Liu, Y., and Luger, R., (2006), “Parametric and Semi-parametric Models of Value-at-Risk: On the Way to Bias Reduction,” Working Paper.

Mayorov, K. (2014), Email correspondence September 5, 2014.

Mignola, G., and Opdyke, J.D., (2012), Email correspondence Regarding Root-Avoiding SLA Based on Numeric Integration Methods, December 2012.

Nešlehová, J., Embrechts, P., and Chavez-Demoulin, V., (2006), "Infinite-Mean Models and the LDA for Operational Risk," *The Journal of Operational Risk*, 1(1), 3-25.

Office of the Comptroller of the Currency, "Guidance on Advanced Approaches GAA 2014-01: Supervisory Guidance for Data, Modeling, and Model Risk Management Under the Operational Risk Advanced Measurement Approaches," GAA 2014 – 1, June 30, 2014.

Opdyke, J.D., (2011), "Robust Statistics vs. MLE for OpRisk Severity Distribution Parameter Estimation (with and without truncation)," presentation at the Operational Risk eXchange (ORX) Analytics Forum, San Francisco, CA, September 27-29.

Opdyke, J.D., (2012), "Better Capital Planning via Exact Sensitivity Analysis Using the Influence Function," presentation at the American Bankers Association: Operational Risk Modeling Forum, Washington DC, July 18-20.

Opdyke, J.D., (2013), "Estimating Operational Risk Capital with Greater Accuracy, Precision, and Robustness … OR … How to Prevent Jensen's Inequality from Inflating Your Operational Risk Capital Estimates," Globally Attended Webinar, February, 2013.

Opdyke, J.D., and Cavallo, A. (2012a), "Estimating Operational Risk Capital: The Challenges of Truncation, The Hazards of Maximum Likelihood Estimation, and the Promise of Robust Statistics," *The Journal of Operational Risk*, Vol. 7, No. 3, 3-90.

Opdyke, J.D., and Cavallo, A. (2012b), "Operational Risk Capital Estimation and Planning: Exact Sensitivity Analyses and Business Decision Making Using the Influence Function," in Operational Risk: New Frontiers Explored, Davis, E., ed., Risk Books, London.

Panjer, H., (1981), "Recursive Evaluation of a Family of Compound Distributions," *ASTIN Bulletin (International Actuarial Association)*, Vol.12(1), 22-26.

Polanski, A., Stoja, E., and Zhang, R., (2013), "Multidimensional Risk and Risk Dependence," Working Paper, http://ssrn.com/abstract=2390107

Reshetar, A., (2008), "Dependence of Operational Losses and the Capital at Risk," Swiss Banking Institute, University of Zurich, available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1081256

Rocco, M., (2011), "Extreme Value Theory for Finance: A Survey," Banca D'Italia, Number 99, July, 2011.

RMA, AMA Group Document, (2011), "Industry Position Paper: Unit of Measure and Dependence," The Risk Management Association (RMA), 2011.

RMA, AMA Group Document, (2013), "Industry Position Paper: AMA Quantification Challenges: AMAG Range of Practice and Observations on 'The Thorny LDA Topics'," The Risk Management Association (RMA), 2013.

Roehr, A., (2002), "Modelling Operational Losses", *Algo Research Quarterly*, Vol.5, No.2, Summer 2002.

Rozenfeld, I., (2011), "Estimating Operational Risk Capital from Truncated Data," ABA Operational Risk Modeling Forum, Charlotte, NC, August 10-12, 2011.

Operational Risk & Regulation, July 2009, "Correlation: Alternative Realities," Incisive Media: Incisive Financial Publishing Limited, Haymarket Hourse, 28-29 Haymarket, London, UK.

Operational Risk & Regulation, June 2013, "OpRisk Europe: 1,000-year Standard is Unreachable, Mignola says," Incisive Media: Incisive Financial Publishing Limited, Haymarket Hourse, 28-29 Haymarket, London, UK.

Operational Risk & Regulation, February 14, 2014, "The Problems of Explaining AMA Volatility," Incisive Media: Incisive Financial Publishing Limited, Haymarket Hourse, 28-29 Haymarket, London, UK.

Sahay, A., Wan, Z., and Keller, B., (2007), "Operational Risk Capital: Asymptotics in the Case of Heavy-Tailed Severity," *The Journal of Operational Risk*, Vol. 2, No. 2, 61-72.

Schaefer, M. (1976), "Note on the *k*-Dimensional Jensen Inequality," *Annals of Probability*, 4(3), pp.502-504.

Schevchenko, P., (2009), "Implementing Loss Distribution Approach for Operational Risk," *Applied Stochastic Models in Business and Industry*, Vol. 26, 227-307.

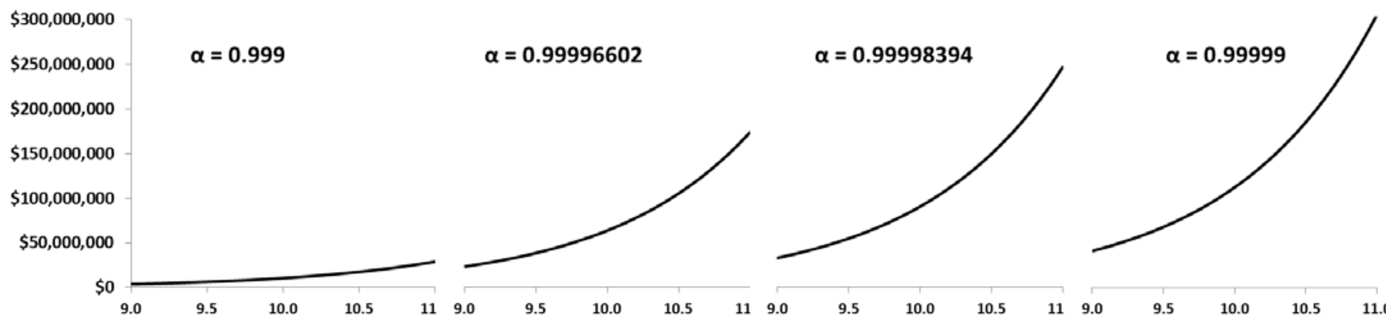
Schevchenko, P., (2011), Modelling Operational Risk Using Bayesian Inference, Springer, New York, New York.

Serfling, R., (2002), "Efficient and Robust Fitting of Lognormal Distributions," *North American Actuarial Journal*, Vol. 6(4).
Smith, J. (1987), "Estimating the Upper Tail of Flood Frequency Distributions," *Water Resources Research*, Vol.23, No.8, 1657-1666.
Taleb, N., Canetti, E., Kinda, T., Loukoianova, E., and Schmieder, C., (2012), "A New Heuristic Measure of Fragility and Tail Risks: Application to Stress Testing," IMF Working Paper, Monetary and Capital Markets Department, WP/12/216.
Taleb, N., and Douady, R., (2013), "Mathematical Definition, Mapping, and Detection of (Anti)Fragility," *Quantitative Finance*, forthcoming.
Taleb, N., (2014), Antifragile: Things that Gain from Disorder, Random House, Inc., New York, New York.
Umande, P., (2013), "A New Approach for Modelling Risk Dependence Using Spatial Point Processes," Working Paper, http://ssrn.com/abstract=2140013.
U.S. Final Rule: Federal Register, Vol. 72, No. 235, 69288-69445.
van Belle, G., (2008), Statistical Rules of Thumb, 2nd Ed., Wiley-Interscience, Hoboken, New Jersey.
Wilde, S., and Grimshaw, S., (2013), "Efficient Computation of Generalized Median Estimators," *Computational Statistics*, Vol. 28(1), 307-317.
Zhou, X., Giacomettib, R., Fabozzie, F., and Tuckera, A., (2013), "Bayesian Estimation of Truncated Data with Applications to Operational Risk Measurement," *Quantitative Finance*, Forthcoming, 1-26.

APPENDIX A

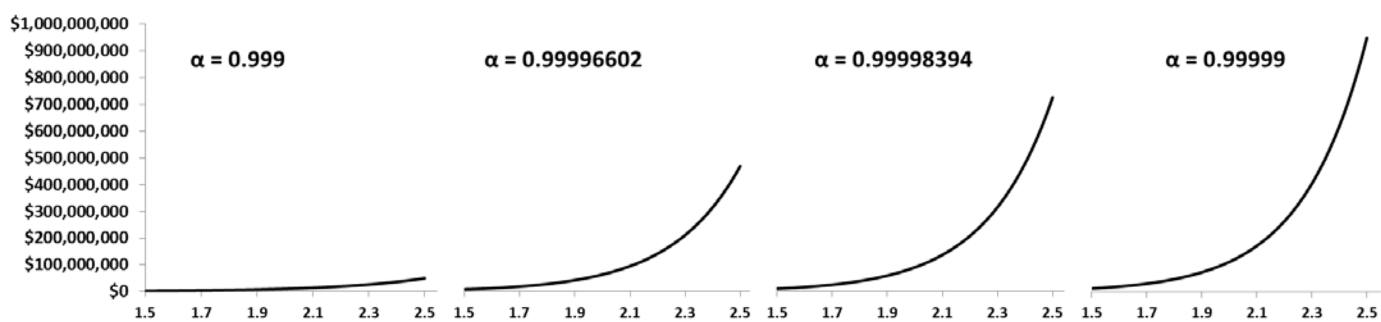
(note that the leftmost graphs in Figure A1, when scaled independently, exhibit notable convexity (except for GPD θ, which is linear))

FIGURE A1: LogNormal Parameter $\boldsymbol{\mu}$ by Quantile by α, Parameter $\boldsymbol{\sigma}$ = 2, Threshold=$0



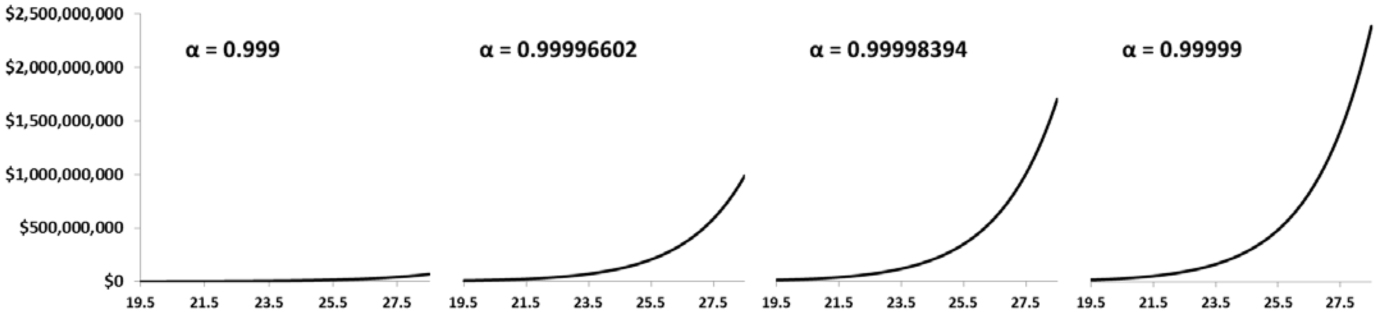
Parameter $\boldsymbol{\mu}$

LogNormal Parameter $\boldsymbol{\sigma}$ by Quantile by α, Parameter $\boldsymbol{\mu}$ = 10, Threshold=$0



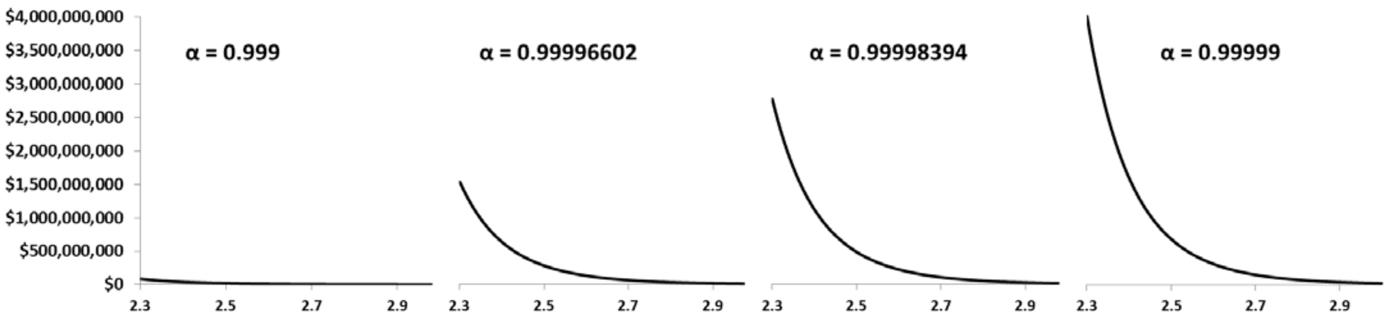
Parameter $\boldsymbol{\sigma}$

LogGamma Parameter ***a*** by Quantile by α, Parameter ***b*** = 2.65, Threshold=$0



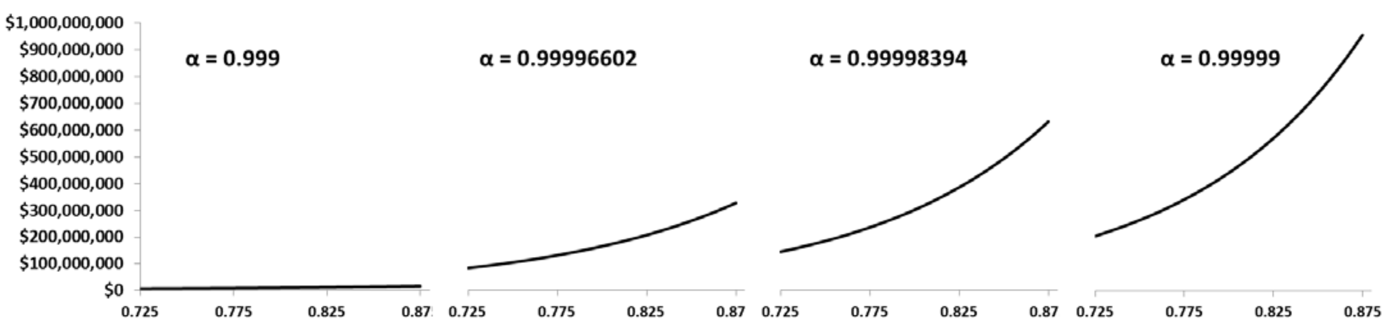
Parameter ***a***

LogGamma, Parameter ***b*** by Quantile by α, Parameter ***a*** = 24, Threshold=$0



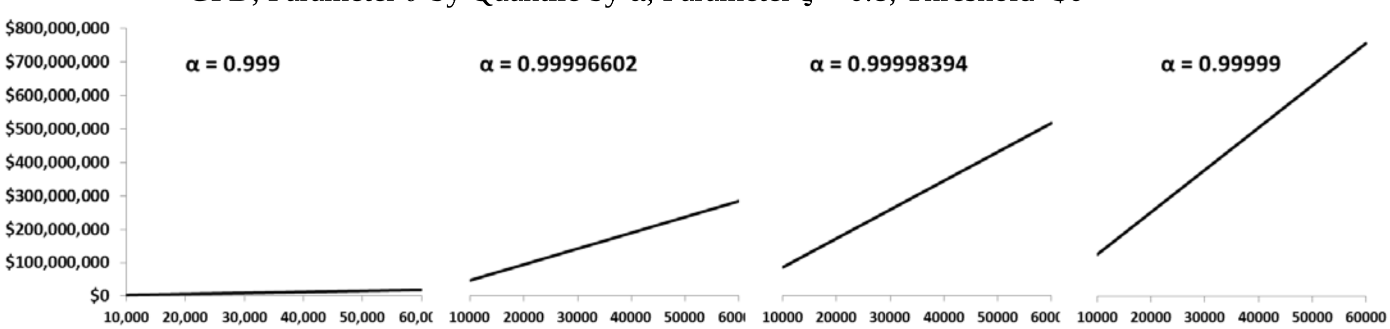
Parameter ***b***

GPD Parameter ξ by Quantile by α, Parameter θ = 35,000, Threshold=$0



Parameter ξ

GPD, Parameter θ by Quantile by α, Parameter ξ = 0.8, Threshold=$0



Parameter θ

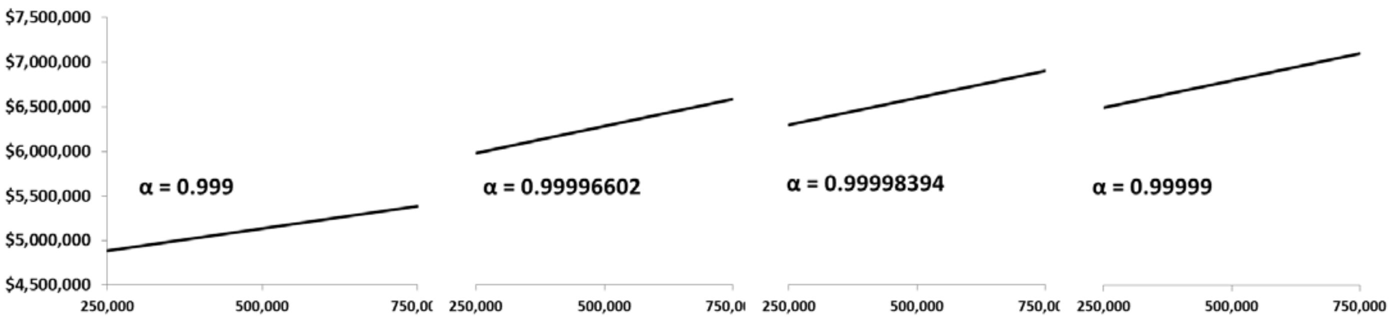
APPENDIX B

TABLE B1: Regulatory and Economic Capital Distributions, 1,000 Simulations λ=25

	GPD Severity (ξ = 1.1, θ = 40,000)		Normal Severity (μ = 500k, σ = 1.5m)	
	RCap (α = 0.999)	ECap (α = 0.9997)	RCap (α = 0.999)	ECap (α = 0.9997)
True Capital	**$2,521,620,617**	**$9,432,295,763**	**$18,916,600**	**$19,336,006**
Mean	$4,606,994,975	$20,895,168,520	$18,890,719	$19,310,226
%Bias	82.70%	121.53%	-0.14%	-0.13%
RMSE	$7,809,076,769	$43,461,854,111	$2,583,208	$2,584,629
StdDev	$7,525,193,727	$41,921,400,048	$2,583,078	$2,584,501
IQR	$3,830,876,336	$16,636,357,991	$3,449,874	$3,459,729
95%CIs	$22,811,712,755	$117,827,228,494	$9,948,476	$9,963,910
Skewness	5.65	6.72	0.14	0.14
Kurtosis	48.80	66.47	0.07	0.06

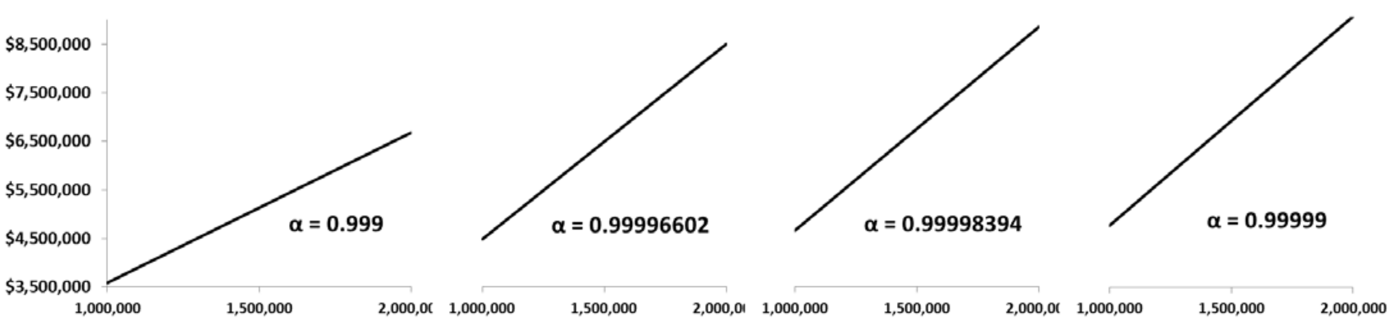
(GPD Graphs ξ = 1.1, θ = 40,000 are virtually identical to GPD graphs in Appendix A above)

FIGURE B1: Normal, Parameter μ by Quantile by α, Parameter σ = 1,500,000, Threshold=$0



Parameter μ

Normal, Parameter σ by Quantile by α, Parameter μ = 500,000, Threshold=$0



Parameter σ

APPENDIX C

TABLE C1: MLE-LDA for Economic and Regulatory Capital Estimation Varying λ Only*

Severity			ECap Bias			RCap Bias		
Dist.	Parm1	Parm2	λ = 15	λ = 25	λ = 100	λ = 15	λ = 25	λ = 100
	μ	**σ**						
LogN	10	2	-1.0%	-0.6%	-0.2%	-1.1%	-0.6%	-0.2%
LogN	7.7	2.55	-1.1%	-0.7%	-0.2%	-1.1%	-0.7%	-0.2%
LogN	10.4	2.5	-1.1%	-0.7%	-0.2%	-1.1%	-0.7%	-0.2%
LogN	9.27	2.77	-1.1%	-0.7%	-0.2%	-1.1%	-0.7%	-0.2%
LogN	10.75	2.7	-1.1%	-0.7%	-0.2%	-1.1%	-0.7%	-0.2%
LogN	9.63	2.97	-1.0%	-0.7%	-0.2%	-1.0%	-0.6%	-0.2%
TLogN	10.2	1.95	-1.0%	-0.6%	-0.2%	-1.0%	-0.6%	-0.2%
TLogN	9	2.2	-1.1%	-0.6%	-0.2%	-1.1%	-0.6%	-0.2%
TLogN	10.7	2.385	-1.1%	-0.7%	-0.2%	-1.1%	-0.7%	-0.2%
TLogN	9.4	2.65	-1.1%	-0.7%	-0.2%	-1.1%	-0.7%	-0.2%
TLogN	11	2.6	-1.1%	-0.7%	-0.2%	-1.1%	-0.7%	-0.2%
TLogN	10	2.8	-1.1%	-0.7%	-0.2%	-1.1%	-0.7%	-0.2%
	a	**b**						
Logg	24	2.65	-1.0%	-0.6%	-0.2%	-1.0%	-0.6%	-0.2%
Logg	33	3.3	-1.0%	-0.6%	-0.2%	-1.0%	-0.6%	-0.2%
Logg	25	2.5	-0.9%	-0.6%	-0.2%	-0.9%	-0.6%	-0.2%
Logg	34.5	3.15	-1.0%	-0.6%	-0.2%	-1.0%	-0.6%	-0.2%
Logg	25.25	2.45	-0.9%	-0.6%	-0.2%	-0.9%	-0.6%	-0.2%
Logg	34.7	3.07	-1.0%	-0.6%	-0.2%	-1.0%	-0.6%	-0.2%
TLogg	23.5	2.65	-1.0%	-0.6%	-0.2%	-1.0%	-0.6%	-0.2%
TLogg	33	3.3	-1.0%	-0.6%	-0.2%	-1.0%	-0.6%	-0.2%
TLogg	24.5	2.5	-1.0%	-0.6%	-0.2%	-0.9%	-0.6%	-0.2%
TLogg	34.5	3.15	-1.0%	-0.6%	-0.2%	-1.0%	-0.6%	-0.2%
TLogg	24.75	2.45	-0.9%	-0.6%	-0.2%	-0.9%	-0.6%	-0.2%
TLogg	34.6	3.07	-1.0%	-0.6%	-0.2%	-1.0%	-0.6%	-0.2%
	ξ	**θ**						
GPD	0.8	35,000	-0.8%	-0.5%	-0.2%	-0.8%	-0.5%	-0.2%
GPD	0.95	7,500	-0.4%	-0.3%	-0.2%	-0.4%	-0.3%	-0.2%
GPD	0.875	47,500	-0.6%	-0.4%	-0.2%	-0.6%	-0.4%	-0.2%
GPD	0.95	25,000	-0.4%	-0.3%	-0.2%	-0.4%	-0.3%	-0.2%
GPD	0.925	50,000	-0.5%	-0.3%	-0.2%	-0.5%	-0.3%	-0.2%
GPD	0.99	27,500	-0.3%	-0.2%	-0.1%	-0.3%	-0.2%	-0.1%
TGPD	0.775	33,500	-0.8%	-0.5%	-0.2%	-0.8%	-0.5%	-0.2%
TGPD	0.8	25,000	-0.8%	-0.5%	-0.2%	-0.8%	-0.5%	-0.2%
TGPD	0.8675	50,000	-0.6%	-0.4%	-0.2%	-0.6%	-0.4%	-0.2%
TGPD	0.91	31,000	-0.5%	-0.4%	-0.2%	-0.5%	-0.4%	-0.2%
TGPD	0.92	47,500	-0.5%	-0.3%	-0.2%	-0.5%	-0.3%	-0.2%
TGPD	0.95	35,000	-0.4%	-0.3%	-0.2%	-0.4%	-0.3%	-0.2%

*NOTE: #simulations = 1,000; α=0.999 and 0.9997 for RCap and ECap, respectively.

APPENDIX D

PDF, CDF, Mean, and Inverse of Fisher information for Six Severities

LogNormal PDF and CDF:

$$f\left(x;\mu,\sigma\right)=\frac{1}{\sqrt{2\pi}\sigma x}e^{-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2} \text{ and } F\left(x;\mu,\sigma\right)=\frac{1}{2}\left[1+erf\left(\frac{\ln(x)-\mu}{\sqrt{2\sigma^2}}\right)\right] \text{ for } 0<x<\infty,\ 0<\sigma<\infty$$

LogNormal Mean:

$$E\left(X\right)=e^{\left(\mu+\sigma^2/2\right)}$$

LogNormal Inverse of Fisher information:

$$A(\theta)^{-1}=\begin{bmatrix}\sigma^2 & 0\\ 0 & \sigma^2/2\end{bmatrix}$$

Truncated LogNormal PDF and CDF:

$$g\left(x;\mu,\sigma\right)=\frac{f\left(x;\mu,\sigma\right)}{1-F\left(H;\mu,\sigma\right)} \text{ and } G\left(x;\mu,\sigma\right)=1-\frac{1-F\left(x;\mu,\sigma\right)}{1-F\left(H;\mu,\sigma\right)} \text{ for } H<x<\infty \text{ and } 0<\sigma<\infty$$

where $f(\ )$ is LogNormal PDF and $F(\ )$ is LogNormal CDF.

Truncated LogNormal Mean:

$$E\left(X\right)=e^{\mu+\sigma^2/2}\cdot\Phi\left(\frac{\mu+\sigma^2-\ln(H)}{\sigma}\right)\cdot\frac{1}{\left[1-F(H)\right]} \text{ where } \Phi(\ ) \text{is the standard normal CDF.}$$

Truncated LogNormal Inverse of Fisher information:

Let $u=\dfrac{\ln(H)-\mu}{\sigma}$, $j=\dfrac{-u^2/2}{\sqrt{2\pi}}$, $J=\dfrac{j}{1-\Phi(u)}$ where Φ is the CDFof the Standard Normal, and

$$INV=\frac{\sigma^2}{\left[2+J\cdot(J-u)\cdot\left(u\cdot(J-u)-3\right)\right]} \text{ then } A(\theta)^{-1}=INV\cdot\begin{bmatrix}2+J\cdot u\cdot\left(1-u\cdot(J-u)\right) & J\cdot\left(u\cdot(J-u)-1\right)\\ J\cdot\left(u\cdot(J-u)-1\right) & 1-\left(J\cdot(J-u)\right)\end{bmatrix}$$

from Roehr (2002) (Note that the first cell of this matrix as presented in Roehr (2002) contains a typo: this is corrected in the presentation above).

Generalized Pareto Distribution (GPD) PDF and CDF:

$$f\left(x;\xi,\theta\right)=\frac{1}{\theta}\left[1+\xi\frac{x}{\theta}\right]^{\left[-\frac{1}{\xi}-1\right]} \text{ and } F\left(x;\xi,\theta\right)=1-\left[1+\xi\frac{x}{\theta}\right]^{\left[-\frac{1}{\xi}\right]}$$

assuming $\xi\geq 0$, for $0\leq x<\infty;\ 0<\theta<\infty$

GPD Mean:

$$E(X)=\frac{\theta}{1-\xi} \text{ for } \xi<1 \ (=\infty \text{ for } \xi \geq 1)$$

GPD Inverse of Fisher information:

$$A(\theta)^{-1}=(1+\xi)\begin{bmatrix}1+\xi & -\theta \\ -\theta & 2\theta^2\end{bmatrix}$$ from Smith (1987).

Truncated GPD PDF and CDF:

$$g(x;\xi,\theta)=\frac{f(x;\xi,\theta)}{1-F(H;\xi,\theta)} \quad \text{and} \quad G(x;\xi,\theta)=1-\frac{1-F(x;\xi,\theta)}{1-F(H;\xi,\theta)}$$

assuming $\xi \geq 0$, for $H \leq x < \infty;\ 0<\theta<\infty$, where $f(\ )$ is GPD PDF and $F(\ )$ is GPD CDF.

Truncated GPD Mean:

$$E(X)=\frac{\theta}{\xi}\cdot\left(\frac{\left[1-F(H)\right]^{-\xi}}{1-\xi}-1\right) \text{ for } \xi<1 \ (=\infty \text{ for } \xi \geq 1)$$

As per Mayorov (2014), this also can be represented as $E(X)=\frac{H+\theta}{1-\xi}$ for $\xi<1$ $(=\infty$ for $\xi \geq 1)$.

Truncated GPD Inverse of Fisher information:

$$A(\theta)^{-1}=(1+\xi)\cdot\begin{bmatrix}(1+\xi) & -\theta\left(1+(1+2\xi)\left(\frac{H}{\theta}\right)\right) \\ -\theta\left(1+(1+2\xi)\left(\frac{H}{\theta}\right)\right) & \theta^2\left(2+2(1+2\xi)\left(\frac{H}{\theta}\right)+(1+\xi)(1+2\xi)\left(\frac{H}{\theta}\right)^2\right)\end{bmatrix}$$ from Roehr (2002).

LogGamma PDF and CDF:[73]

$$f(x;a,b)=\frac{b^a\left(\log(x)\right)^{(a-1)}}{\Gamma(a)x^{b+1}} \quad \text{and} \quad F(x;a,b)=\int_1^x\frac{b^a\left(\log(y)\right)^{(a-1)}}{\Gamma(a)y^{b+1}}dy$$ for $1 \leq x < \infty;\ 0<a;\ 0<b$ where $\Gamma(a)$

is the complete gamma function.The domain can be changed to $\mu \leq x < \infty$ if a location parameter, μ, is added and $x-\mu$ is substituted for x (so if μ = 1, the domain would range from zero and approach positive infinity).

LogGamma Mean:

$$E(X)=\left(\frac{b}{b-1}\right)^a \text{ for } b>1 \ (=\infty \text{ for } b \leq 1)$$

LogGamma Inverse of Fisher information:

[73] Note that this parameterization of the two-parameter LogGamma is the inverted parameterization (b = 1/b).

$$A(\theta)^{-1} = \frac{1}{\left(a/b^2\right)\cdot trigamma\left(a\right) - 1/b^2}\begin{bmatrix} a/b^2 & 1/b \\ 1/b & trigamma\left(a\right)\end{bmatrix}$$ from Opdyke and Cavallo (2012a).[74]

Truncated LogGamma PDF and CDF:

$$g\left(x;a,b\right) = \frac{f\left(x;a,b\right)}{1-F\left(H;a,b\right)} \quad \text{and} \quad G\left(x;a,b\right) = 1 - \frac{1-F\left(x;a,b\right)}{1-F\left(H;a,b\right)}$$

for $1 \le x < \infty;\ 0 < a;\ 0 < b$, where $f(\ )$ is LogGamma PDF and $F(\ )$ is LogGamma CDF.

Truncated LogGamma Mean:

$$E\left(X\right) = \left(\frac{b}{b-1}\right)^a \cdot \frac{1-J\left(\log\left(H\right)\left(b-1\right);a,1\right)}{\left[1-F\left(H\right)\right]}$$ for $b > 1$, $= \infty$ for $b \le 1$, where $J(\ )$ is the CDF of the Gamma distribution

Although Kim (2010) presents a derivation of the conditional (tail) mean for the LogGamma with direct parameterization (as opposed to LogGamma with inverted (1/b) parameterization as shown above and as used in this paper), the above mean for the Truncated LogGamma does not appear to have been presented in the literature previously and its derivation is shown below. It is known that for *J*( ) Gamma CDF,

$$1 - J\left(x;a,b=1\right) = \frac{\Gamma\left(a,x\right)}{\Gamma\left(a\right)}$$ where $\Gamma\left(a,x\right)$ is the upper incomplete gamma function and $\Gamma(\ )$ is the complete gamma function.

Also, the tail mean (i.e. mean beyond the threshold, *H*, as opposed to the mean of the truncated distribution) of

the LogGamma is $TM\left(H\right) = \int_H^\infty y \cdot f\left(y;a,b\right)dy = \left(\frac{b}{b-1}\right)^a \cdot \frac{\Gamma\left(a, Log\left(H\right)\left(b-1\right)\right)}{\Gamma\left(a\right)}$

Therefore, because Mean of Truncated Distribution = *TM*(*H*) / (1-*F*(*H*)),

Mean of Truncated LogGamma = $\left(\frac{b}{b-1}\right)^a \cdot \left[1-J\left(\log\left(H\right)\left(b-1\right);a,1\right)\right] \cdot \frac{1}{1-F\left(H\right)}$

As per Mayorov (2014), this also can be represented as $\left(\frac{b}{b-1}\right)^a \cdot \frac{1-F\left(H;a,b-1\right)}{1-F\left(H;a,b\right)}$.

Truncated LogGamma Inverse of Fisher information:

$$A(\theta)^{-1} = \begin{bmatrix} A & B \\ B & D\end{bmatrix}^{-1}$$ where

$$A = trigamma\left(a\right) - \frac{\left[\int_{1^+}^{H} \ln\left(b\right) + \ln\left(\ln\left(x\right)\right) - digamma\left(a\right) f\left(x\right)dx\right]^2}{\left[1-F\left(H;a,b\right)\right]^2}$$

[74] The digamma and trigamma functions are the first and second order logarithmic derivatives of the complete gamma function: *digamma*(z) = *d*/*dz* ln [Γ(z)] and *trigamma*(z) = d^2/dz^2 ln [Γ(z)].

$$-\frac{\left[1-F\left(H;a,b\right)\right]\cdot\int_{1^+}^{H}\left[\ln\left(b\right)+\ln\left(\ln\left(x\right)\right)-digamma\left(a\right)\right]^2-trigamma\left(a\right)f\left(x\right)dx}{\left[1-F\left(H;a,b\right)\right]^2}$$

$$B=-\frac{1}{b}-\frac{\left[1-F\left(H;a,b\right)\right]\cdot\frac{1}{b}\cdot F\left(H;a,b\right)}{\left[1-F\left(H;a,b\right)\right]^2}$$

$$-\frac{\left[1-F\left(H;a,b\right)\right]\cdot\int_{1^+}^{H}\left[\ln\left(b\right)+\ln\left(\ln\left(x\right)\right)-digamma\left(a\right)\right]\cdot\left[\frac{a}{b}-\ln\left(x\right)\right]f\left(x\right)dx}{\left[1-F\left(H;a,b\right)\right]^2}$$

$$-\frac{\int_{1^+}^{H}\ln\left(b\right)+\ln\left(\ln\left(x\right)\right)-digamma\left(a\right)f\left(x\right)dx\cdot\int_{1^+}^{H}\left(\frac{a}{b}-\ln\left(x\right)\right)f\left(x\right)dx}{\left[1-F\left(H;a,b\right)\right]^2}$$

$$D=\frac{a}{b^2}-\frac{\left[\int_{1^+}^{H}\left(\frac{a}{b}-\ln\left(y\right)\right)f\left(x\right)dx\right]^2+\left[1-F\left(H;a,b\right)\right]\cdot\int_{1^+}^{H}\frac{a\left(a-1\right)}{b^2}-\frac{2a\ln\left(y\right)}{b}+\left[\ln\left(y\right)\right]^2f\left(x\right)dx}{\left[1-F\left(H;a,b\right)\right]^2}$$

from Opdyke and Cavallo (2012a).[75]

Zhou (2013) presents the equivalent of the above for the direct parameterization of the LogGamma,, but this, too, requires computationally expensive numeric integration.

Below I present an analytic approximation for the Fisher information of the Truncated LogGamma (inverted parameterization) that avoids numeric integration, and as a consequence, is over seven times faster to implement. For applications in this paper, the accuracy of the matrix terms, using η = 0.001 (described below), ranges from eight to eleven decimal places, which is very sufficient for purposes of estimating operational risk capital (that is to say, no more than a few thousand dollars divergent from estimated capital that is hundreds of millions, or even billions of dollars).

$$A\left(\theta\right)^{-1}=\begin{bmatrix}A & B\\ B & D\end{bmatrix}^{-1}$$ where

[75] Note that the domain of the threshold, H, is $1\le H<\infty$ instead of $0\le H<\infty$ to avoid having to include a third location parameter for the LogGamma. The numeric effects of this mathematical convenience are de minimis, if even calculable. The same convention is used throughout the derivation of the analytic approximation for the Fisher information of the Truncated LogGamma, where the threshold is labeled "t" rather than H.

$$A = -\frac{MeijerG[\{\{\},\{1,1\}\},\{\{0,0,a\},\{\}\},bLog[t]]^2 - 2\Gamma[a,bLog[t]]MeijerG[\{\{\},\{1,1,1\}\},\{\{0,0,0,a\},\{\}\},bLog[t]]}{\Gamma[a,bLog[t]]^2}$$

$$B = \frac{-1+\dfrac{t^{-b}MeijerG[\{\{\},\{1+a,1+a\}\},\{\{a,a,2a\},\{\}\},bLog[t]]}{\Gamma[a,bLog[t]]^2}}{b}$$

$$D = t^{-3b}(1+(-1+a)t^b ExpIntegralE[2-a,bLog[t]])\times$$

$$\times\frac{(-1+t^b ExpIntegralE[1-a,bLog[t]](1-a+at^b ExpIntegralE[1-a,bLog[t]]+bLog[t]))}{b^3 ExpIntegralE[1-a,bLog[t]]^3 Log[t]}$$

where

$\Gamma\left(a,x\right)$ is the upper incomplete gamma function,

$MeijerG[\{\{a1,...,an\},\{a\left(n+1\right),...,ap\}\},\{\{b1,...,bm\},\{b\left(m+1\right),...,bq\}\},z] =$

$$= \frac{1}{2\pi i}\int_{\gamma L}\frac{\prod_{j=1}^{m}\Gamma\left(b_j+s\right)\prod_{j=1}^{m}\Gamma\left(1-a_j-s\right)}{\prod_{j=n+1}^{p}\Gamma\left(a_j+s\right)\prod_{j=m+1}^{q}\Gamma\left(1-b_j-s\right)}x^{-s}ds$$

, and under certain conditions, in terms of generalized hypergeometric functions

$$= \sum_{h=1}^{m}\frac{\prod_{j=1}^{m}\Gamma\left(b_j-b_h\right)*\prod_{j=1}^{n}\Gamma\left(1+b_h-a_j\right)z^{b_h}}{\prod_{j=m+1}^{q}\Gamma\left(1+b_h-b_j\right)\prod_{j=n+1}^{p}\Gamma\left(a_j-b_h\right)}\times {}_pF_{q-1}\left(\begin{matrix}1+b_h-\mathrm{a}_\mathrm{p} \\ \left(1+b_h-\mathrm{a}_\mathrm{p}\right)^*\end{matrix}\middle|\left(-1\right)^{p-m-n}z\right)$$

and

$$ExpIntegralE[n,z] = \int_1^\infty \frac{e^{-zt}}{t^n}dt = z^{n-1}\Gamma\left(1-n,z\right)$$

Unfortunately, the MeijerG[ ] function converges too slowly, when used with values relevant to this setting, to be used by non-symbolic programming languages, that is, all statistical and computer programming languages.[76] In other words, the terms of the MeijerG[ ] become too small (i.e. less than 10E-16) to calculate for the computer chips on most modern computers, but we cannot discount these terms as they are not rapidly

[76] Mathematica is the major symbolic programming language that can correctly execute such calculations, but it is not as useable as the major statistical programming languages for nontrivial volumes of empirical statistical analyses, as are required for operational risk capital estimation. So these calculations must be put into terms that all major statistical programming languages, and most modern computer chips, can successfully calculate with sufficient precision.

divergent and notably affect the final result of the calculation. To circumvent this obstacle to practical usage, $A(\theta)^{-1}$ can be expanded as below:

$$A = \frac{1}{a^4\left[\Gamma\left(a,bLog[t]\right)\right]^2} \times$$

$$\times\Big\{\Big[-\left(GHG\left(\{a,a\},\{a+1,a+1\};-bLog[t]\right)\right)^2\Big]\cdot\left(bLog[t]\right)^{2a}$$

$$+2a\left(bLog[t]\right)^a\cdot\Big[-\Gamma\left(a,bLog[t]\right)\cdot GHG\left(\{a,a,a\},\{a+1,a+1,a+1\};-bLog[t]\right)$$

$$+a\Gamma(a)\cdot GHG\left(\{a,a\},\{a+1,a+1\};-bLog[t]\right)\cdot\left(Log\left(bLog[t]\right)-digamma(a)\right)\Big]$$

$$+a^4\Gamma(a)\Big[-\left(\Gamma(a)-\Gamma\left(a,bLog[t]\right)\right)\cdot\left(Log\left(bLog[t]\right)-digamma(a)\right)^2+\Gamma\left(a,bLog[t]\right)\cdot trigamma(a)\Big]\Big\}$$

$$B = \frac{1}{a^2b\left[\Gamma\left(a,bLog[t]\right)\right]^2} \times$$

$$\Big\{t^{-b}\cdot GHG\left(\{a,a\},\{a+1,a+1\};-bLog[t]\right)\cdot\left(bLog[t]\right)^{2a}$$

$$-a^2\left(t^b\left[\Gamma\left(a,bLog[t]\right)\right]^2+\Gamma(a)\left(bLog[t]\right)^a\left(Log\left(bLog[t]\right)-digamma(a)\right)\right)\Big\}$$

$$D = \frac{1}{b^3Log[t]\left[\Gamma\left(a,bLog[t]\right)\right]^3}\times\Big\{\Big[t^{-3b}\Gamma\left(a-1,bLog[t]\right)Log[t]+\left(bLog[t]\right)^a\Big]\times$$

$$\times\Big[at^{2b}\left[\Gamma\left(a,bLog[t]\right)\right]^2-\left(bLog[t]\right)^{2a}+t^{2b}\Gamma\left(a,bLog[t]\right)\left(bLog[t]\right)^{2a}\left(1-a+bLog[t]\right)\Big]\Big\}$$

which reduces to

$$D = \frac{a}{b^2}+\frac{t^{-b}\left(bLog[t]\right)^a\left(1-a+bLog[t]\right)}{b^2\Gamma\left(a,bLog[t]\right)}-\frac{t^{-2b}\left(bLog[t]\right)^{2a}}{b^2\left[\Gamma\left(a,bLog[t]\right)\right]^2}$$

where

$$GHG\left(\{a1,...,ap\},\{b1,...,bq\};z\right)=\sum_{n=0}^{\infty}\frac{(a1)_n...(ap)_n}{(b1)_n...(bq)_n}\frac{z^n}{n!},$$

where $(a)_n = a(a+1)(a+2)...(a+n-1), (a)_0 = 1$, is the generalized hypergeometric function.

Unfortunately, the generalized hypergeometric function suffers from the same usability challenge as does the MeijerG[ ] function: the terms become too small (i.e. less than 10E-16) to calculate for the computer chips on most modern computers, but these terms are material to the final result as they are not rapidly divergent. Fortunately, identities for the generalized hypergeometric function exist that contain terms similar to those found in the Fisher information above. For example,

$$GHG\left(\{a1,...,ap\},\{a1+1,...,ap+1\};z\right)=\sum_{k=1}^{q}\left[GHG(ak,ak+1;z)\cdot\prod_{j=1,\,j\neq k}^{q}\frac{a_j}{a_j-a_k}\right]$$

and

$$GHG(a,a+1;z)=\Gamma(a+1)(-z)^{-a}\left(1-\frac{\Gamma(a,-z)}{\Gamma(a)}\right)$$

The only problem with using these two identities to provide the generalized hypergeometric function in terms that can be readily calculated using most computer hardware and software (that is, non-symbolic programming languages) is that in our case, a1 = a2 = … = ap = a, so the last product term is undefined (i.e. $\frac{a_j}{a_j-a_j}=\frac{a_j}{0}\sim undefined$). Consequently, I use these identities to approximate the Fisher information by increasing and decreasing values for a1 and a2 and a3 by a small amount, η (=0.001). This yields values of the elements of the Fisher information that deviate from true values by between eight and eleven decimal places, which as mentioned above, is very sufficient for purposes of estimating operational risk capital (that is to say, no more than a few thousand dollars divergent from estimated capital that is hundreds of millions, or even billions of dollars).

Also, we saw in deriving the mean of the truncated LogGamma severity above that the upper incomplete gamma function $\Gamma(s,x)=\Gamma(s)\cdot\left(1-J(x;s,b=1)\right)$ where *J*( ) is the CDF of the Gamma distribution, so this can be used to provide the upper incomplete gamma function in readily calculable terms.

Substituting both of the above term changes into the Fisher information, let

a = parameter 1; b = parameter 2; t = threshold; $\eta = 0.001$; $adown = a - \eta$; $aup = a + \eta$; $z = -bLog[t]$

divide $a = diva = \frac{\Gamma(a+1)}{(-z)^{a}}$; divide $adown = divad = \frac{\Gamma(adown+1)}{(-z)^{adown}}$; divide $aup = divau = \frac{\Gamma(aup+1)}{(-z)^{aup}}$

so

$$GHG2 = divad \cdot J(-z; adown, 1)\frac{aup}{aup - adown} + divau \cdot J(-z; aup, 1)\frac{adown}{adown - aup}$$

$$GHG3 = divad \cdot J(-z; adown, 1)\left(\frac{aup}{aup - adown}\right)\left(\frac{a}{a - adown}\right) + diva \cdot J(-z; a, 1)\left(\frac{adown}{adown - a}\right)\left(\frac{aup}{aup - a}\right)$$

$$+ divau \cdot J(-z; aup, 1)\left(\frac{adown}{adown - aup}\right)\left(\frac{a}{a - adown}\right)$$ where $J(\,)$ is the CDF of the Gamma distribution.

UIG = upper incomplete gamma function $= \Gamma(a, -z) = \Gamma(a)\left(1 - J(-z; a, b=1)\right)$, then

$$A(\theta)^{-1} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1}$$ where

$$A = \frac{1}{a^4 UIG^2} \times \Big\{ \left[-(GHG2)^2\right] \cdot (-z)^{2a} + 2a(-z)^{a} \cdot \left[-UIG \cdot GHG3 + a\Gamma(a) \cdot GHG2 \cdot \left(Log(-z) - digamma(a)\right)\right]$$
$$+ a^4\Gamma(a)\left[-\left(\Gamma(a) - UIG\right) \cdot \left(Log(-z) - digamma(a)\right)^2 + UIG \cdot trigamma(a)\right] \Big\}$$

$$B = \frac{1}{a^2 b UIG^2} \times \left\{ t^{-b} \cdot GHG2 \cdot (-z)^{2a} - a^2\left(t^{b} UIG^2 + \Gamma(a)(-z)^{a}\left(Log(-z) - digamma(a)\right)\right) \right\}$$

$$D = \frac{a}{b^2} + \frac{t^{-b}(-z)^{a}(1 - a - z)}{b^2 UIG} - \frac{t^{-2b}(-z)^{2a}}{b^2 UIG^2}$$

And now all terms use functions that are readily calculable using any computer and statistical and/or programming language. Note again that due to the use of η, this is an analytical approximation to the Fisher information for the Truncated LogGamma distribution (the first known to this author), but one that is sufficiently accurate for the purpose of estimating operational risk capital with deviations of, at most, a few thousand dollars for capital estimates of hundreds of millions, if not billions of dollars. And when compared to estimation that relies on numeric integration, the speed increases provided by this approximation approach an order of magnitude for the values used in this study.

APPENDIX E

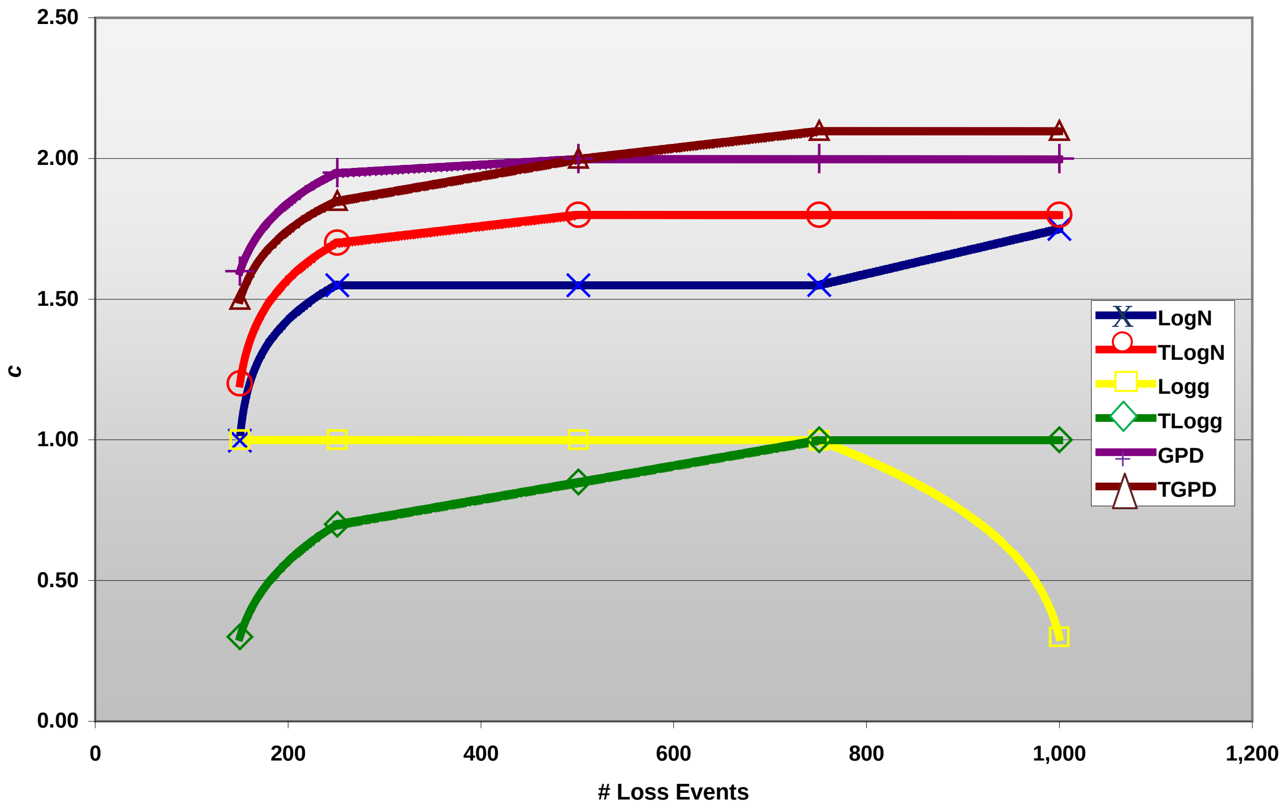
TABLE E1: Values of *c*(*sev*, *n*) by Severity by # of Loss Events

Severity	150	250	500	750	1000	Root
LogN	1.00	1.55	1.55	1.55	1.75	8
TLogN	1.20	1.70	1.80	1.80	1.80	8
Logg	1.00	1.00	1.00	1.00	0.30	3
TLogg	0.30	0.70	0.85	1.00	1.00	3
GPD	1.60	1.95	2.00	2.00	2.00	10
TGPD	1.50	1.85	2.00	2.10	2.10	10

FIGURE E1:

Values of *c*(*sev*, *n*) by Severity by # of Loss Events

(Linear and Non-Linear Interpolation via (5) with Roots Specified Above for Shaded Ranges)



APPENDIX F

Available at http:///www.risk.net/statis/about-the-journal-operational-risk and at http://www.DataMineit.com and from the author upon request (JDOpdyke@DataMineit.com).

TABLE F4a

RCE vs. LDA for Regulatory Capital Estimation Under iid ($m, λ=25)*

Severity Dist.	Parm1	Parm2	True RCap	Mean MLE RCap	MLE Bias	MLE Bias%	Mean RCE RCap	RCE Bias	RCE Bias%	RMSE MLE RCap	RMSE RCE RCap	RMSE RCap RCE/MLE	StdDev MLE RCap	StdDev RCE RCap	StdDev RCap RCE/MLE	95%CIs MLE RCap	95%CIs RCE RCap	95%CIs RCap RCE/MLE	CV MLE	CV RCE	IQR MLE RCap	IQR RCE RCap	IQR RCap RCE/MLE	Skew MLE RCap	Skew RCE RCap	Kurtosis MLE RCap	Kurtosis RCE RCap
	μ	σ																									
LogN	10	2	$63	$67	$4	6.7%	$63	$0	0.5%	$25	$23	91.8%	$25	$23	93.1%	$101	$93	92.7%	0.373	0.368	$30	$29	94.5%	1.11	1.09	1.95	1.86
LogN	7.7	2.55	$53	$59	$6	11.5%	$54	$1	1.5%	$30	$26	87.7%	$29	$26	89.6%	$117	$104	89.4%	0.494	0.487	$33	$30	91.4%	1.48	1.44	3.58	3.36
LogN	10.4	2.5	$649	$720	$72	11.0%	$658	$9	1.4%	$355	$313	88.1%	$348	$313	89.9%	$1,390	$1,248	89.8%	0.483	0.476	$399	$366	91.7%	1.44	1.41	3.41	3.20
LogN	9.27	2.77	$603	$686	$83	13.8%	$614	$12	2.0%	$382	$328	86.0%	$373	$328	88.0%	$1,481	$1,302	87.9%	0.543	0.534	$414	$372	89.9%	1.64	1.59	4.43	4.12
LogN	10.75	2.7	$2,012	$2,275	$263	13.1%	$2,048	$37	1.8%	$1,229	$1,063	86.5%	$1,200	$1,063	88.5%	$4,778	$4,223	88.4%	0.528	0.519	$1,345	$1,215	90.4%	1.59	1.55	4.15	3.87
LogN	9.63	2.97	$1,893	$2,198	$305	16.1%	$1,939	$46	2.4%	$1,329	$1,121	84.3%	$1,294	$1,120	86.6%	$5,125	$4,430	86.4%	0.589	0.578	$1,401	$1,240	88.5%	1.79	1.73	5.33	4.90
TLogN	10.2	1.95	$76	$85	$9	11.5%	$75	-$1	-1.8%	$52	$41	79.4%	$51	$41	80.5%	$179	$146	81.6%	0.598	0.547	$48	$41	86.3%	2.97	2.52	17.44	12.57
TLogN	9	2.2	$76	$96	$20	26.5%	$75	-$1	-1.4%	$88	$50	56.9%	$86	$50	58.4%	$254	$173	68.2%	0.899	0.673	$68	$50	73.8%	4.89	2.49	41.21	10.26
TLogN	10.7	2.385	$670	$847	$177	26.4%	$700	$30	4.5%	$665	$469	70.5%	$641	$468	73.0%	$2,288	$1,755	76.7%	0.757	0.670	$576	$460	79.7%	3.41	2.44	24.05	10.90
TLogN	9.4	2.65	$643	$894	$251	39.1%	$628	-$14	-2.2%	$1,087	$536	49.3%	$1,057	$536	50.7%	$3,108	$1,851	59.6%	1.183	0.853	$659	$470	71.3%	5.39	3.19	43.88	17.16
TLogN	11	2.6	$2,085	$2,651	$566	27.1%	$2,123	$38	1.8%	$2,568	$1,771	69.0%	$2,504	$1,771	70.7%	$8,801	$6,308	71.7%	0.945	0.834	$1,781	$1,407	79.0%	4.20	3.56	27.45	20.10
TLogN	10	2.8	$1,956	$2,743	$787	40.2%	$1,965	$9	0.5%	$3,033	$1,694	55.9%	$2,929	$1,694	57.9%	$10,400	$6,458	62.1%	1.068	0.862	$2,150	$1,469	68.3%	3.83	2.63	21.70	9.88
	a	b																									
Logg	24	2.65	$85	$97	$12	13.6%	$87	$2	2.1%	$62	$53	86.0%	$61	$53	87.5%	$217	$191	88.0%	0.628	0.612	$63	$56	88.9%	2.67	2.57	13.66	12.74
Logg	33	3.3	$100	$108	$8	8.5%	$99	$0	-0.4%	$56	$50	89.0%	$56	$50	90.0%	$210	$190	90.2%	0.517	0.507	$60	$55	90.9%	1.80	1.73	6.34	5.70
Logg	25	2.5	$444	$513	$70	15.7%	$455	$11	2.5%	$355	$301	84.8%	$348	$301	86.5%	$1,241	$1,069	86.2%	0.678	0.662	$356	$308	86.7%	2.28	2.21	8.72	8.19
Logg	34.5	3.15	$448	$497	$49	10.9%	$452	$4	0.8%	$296	$260	87.6%	$292	$260	88.8%	$1,032	$920	89.1%	0.589	0.575	$286	$258	90.3%	2.65	2.56	12.36	11.55
Logg	25.25	2.45	$766	$906	$140	18.3%	$799	$32	4.2%	$647	$543	83.9%	$632	$542	85.8%	$2,277	$1,909	83.8%	0.697	0.679	$654	$576	88.1%	2.60	2.49	13.53	12.38
Logg	34.7	3.07	$818	$930	$112	13.7%	$841	$23	2.8%	$589	$510	86.6%	$578	$510	88.1%	$2,043	$1,807	88.5%	0.622	0.606	$571	$508	88.9%	2.74	2.63	13.55	12.52
TLogg	23.5	2.65	$124	$193	$70	56.1%	$137	$13	10.8%	$273	$181	66.4%	$264	$181	68.5%	$830	$497	59.9%	1.365	1.317	$151	$100	66.6%	5.49	6.34	46.55	62.21
TLogg	33	3.3	$130	$174	$44	34.1%	$130	$0	-0.1%	$173	$93	53.9%	$167	$93	55.8%	$573	$347	60.5%	0.964	0.721	$128	$91	70.9%	3.69	2.53	20.94	10.23
TLogg	24.5	2.5	$495	$794	$299	60.4%	$516	$20	4.1%	$1,103	$554	50.2%	$1,062	$554	52.1%	$3,610	$1,957	54.2%	1.337	1.074	$610	$385	63.2%	4.16	4.04	23.92	24.12
TLogg	34.5	3.15	$510	$635	$125	24.5%	$539	$29	5.8%	$544	$397	73.1%	$529	$396	74.9%	$1,894	$1,527	80.6%	0.834	0.736	$495	$411	83.1%	3.73	2.52	29.80	12.36
TLogg	24.75	2.45	$801	$1,305	$504	62.9%	$848	$47	5.9%	$1,938	$916	47.3%	$1,871	$915	48.9%	$6,376	$2,809	44.0%	1.434	1.079	$979	$636	65.0%	5.63	5.09	49.92	41.76
TLogg	34.6	3.07	$867	$1,078	$211	24.3%	$925	$58	6.7%	$927	$709	76.5%	$902	$707	78.3%	$3,235	$2,517	77.8%	0.838	0.764	$771	$645	83.7%	3.04	2.61	15.19	11.16
	ξ	θ																									
GPD	0.8	35,000	$149	$233	$85	56.9%	$152	$3	2.2%	$295	$167	56.7%	$282	$167	59.2%	$973	$579	59.5%	1.210	1.099	$190	$123	64.8%	3.67	3.39	20.04	17.32
GPD	0.95	7,500	$121	$212	$91	75.6%	$124	$3	2.7%	$311	$156	50.3%	$297	$156	52.6%	$971	$529	54.4%	1.399	1.260	$182	$104	57.3%	4.31	3.93	28.14	24.03
GPD	0.875	47,500	$391	$640	$249	63.7%	$396	$5	1.2%	$870	$466	53.6%	$834	$466	55.9%	$2,753	$1,555	56.5%	1.302	1.177	$547	$329	60.2%	4.04	3.69	24.67	21.04
GPD	0.95	25,000	$403	$697	$295	73.2%	$408	$5	1.3%	$1,019	$513	50.3%	$976	$513	52.6%	$3,139	$1,750	55.8%	1.399	1.258	$594	$346	58.2%	4.41	4.01	29.64	25.31
GPD	0.925	50,000	$643	$1,079	$436	67.8%	$645	$1	0.2%	$1,535	$792	51.6%	$1,472	$792	53.8%	$4,845	$2,715	56.0%	1.364	1.228	$931	$543	58.3%	4.37	3.96	29.24	24.68
GPD	0.99	27,500	$636	$1,121	$486	76.4%	$637	$2	0.3%	$1,698	$828	48.8%	$1,627	$828	50.9%	$5,131	$2,781	54.2%	1.451	1.300	$964	$539	55.9%	4.72	4.29	34.30	29.47
TGPD	0.775	33,500	$141	$214	$73	52.0%	$144	$3	2.2%	$297	$170	57.4%	$288	$170	59.2%	$806	$504	62.5%	1.341	1.181	$178	$118	65.9%	7.53	6.40	102.72	77.40
TGPD	0.8	25,000	$140	$220	$80	56.9%	$145	$5	3.3%	$315	$179	56.8%	$305	$179	58.7%	$992	$591	59.5%	1.383	1.233	$177	$113	64.0%	5.99	5.55	57.29	51.15
TGPD	0.8675	50,000	$452	$737	$285	63.0%	$466	$13	3.0%	$1,062	$576	54.3%	$1,023	$576	56.3%	$3,309	$1,935	58.5%	1.387	1.237	$631	$396	62.8%	4.61	4.11	31.26	25.42
TGPD	0.91	31,000	$451	$761	$309	68.6%	$463	$12	2.7%	$1,174	$603	51.4%	$1,132	$603	53.3%	$3,362	$1,889	56.2%	1.489	1.302	$646	$392	60.7%	5.34	4.52	43.26	31.44
TGPD	0.92	47,500	$698	$1,149	$451	64.7%	$704	$7	0.9%	$1,668	$888	53.2%	$1,606	$888	55.3%	$5,192	$2,946	56.7%	1.397	1.261	$955	$580	60.7%	4.33	3.92	26.32	21.45
TGPD	0.95	35,000	$717	$1,206	$489	68.2%	$715	-$2	-0.2%	$2,009	$991	49.3%	$1,948	$991	50.9%	$5,307	$2,972	56.0%	1.615	1.386	$1,023	$613	60.0%	7.57	6.14	98.27	67.33

*NOTE: #simulations = 1,000; λ = 25 for 10 years so n ~ 250; α=0.999

TABLE F4b

RCE vs. LDA for Economic Capital Estimation Under iid ($m, λ=25)*

Severity Dist.	Parm1	Parm2	True ECap	Mean MLE ECap	MLE Bias	MLE Bias%	Mean RCE ECap	RCE Bias	RCE Bias%	RMSE MLE ECap	RMSE RCE ECap	RMSE ECap RCE/MLE	StdDev MLE ECap	StdDev RCE ECap	StdDev ECap RCE/MLE	95%CIs MLE ECap	95%CIs RCE ECap	95%CIs ECap RCE/MLE	CV MLE	CV RCE	IQR MLE ECap	IQR RCE ECap	IQR ECap RCE/MLE	Skew MLE ECap	Skew RCE ECap	Kurtosis MLE ECap	Kurtosis RCE ECap
	μ	σ																									
LogN	10	2	$107	$115	$8	7.8%	$108	$1	1.1%	$47	$43	91.3%	$46	$43	92.7%	$186	$172	92.3%	0.402	0.397	$56	$51	92.3%	1.20	1.18	2.35	2.25
LogN	7.7	2.55	$107	$121	$14	13.2%	$109	$3	2.5%	$66	$57	87.0%	$64	$57	89.0%	$255	$227	88.9%	0.531	0.522	$72	$64	89.1%	1.61	1.57	4.34	4.05
LogN	10.4	2.5	$1,286	$1,449	$163	12.7%	$1,316	$30	2.4%	$769	$673	87.4%	$752	$672	89.4%	$2,998	$2,675	89.2%	0.519	0.511	$854	$764	89.4%	1.57	1.53	4.12	3.85
LogN	9.27	2.77	$1,293	$1,498	$205	15.8%	$1,333	$40	3.1%	$898	$764	85.2%	$874	$763	87.4%	$3,465	$3,024	87.3%	0.583	0.573	$958	$839	87.6%	1.78	1.73	5.39	4.98
LogN	10.75	2.7	$4,230	$4,864	$634	15.0%	$4,352	$123	2.9%	$2,828	$2,425	85.8%	$2,756	$2,422	87.9%	$10,945	$9,609	87.8%	0.567	0.557	$3,050	$2,687	88.1%	1.73	1.68	5.04	4.67
LogN	9.63	2.97	$4,303	$5,097	$794	18.5%	$4,461	$158	3.7%	$3,321	$2,769	83.4%	$3,224	$2,765	85.8%	$12,725	$10,912	85.8%	0.632	0.620	$3,434	$2,958	86.1%	1.95	1.89	6.52	5.95
TLogN	10.2	1.95	$126	$144	$18	14.7%	$124	-$2	-1.3%	$101	$77	76.1%	$99	$77	77.4%	$346	$266	76.9%	0.687	0.618	$88	$73	83.7%	3.49	2.89	23.76	16.25
TLogN	9	2.2	$133	$179	$46	35.0%	$131	-$2	-1.5%	$202	$97	48.0%	$197	$97	49.4%	$537	$335	62.3%	1.096	0.741	$135	$96	70.8%	6.19	2.51	63.11	9.95
TLogN	10.7	2.385	$1,267	$1,678	$411	32.4%	$1,338	$71	5.6%	$1,521	$1,003	65.9%	$1,464	$1,000	68.3%	$5,074	$3,663	72.2%	0.873	0.747	$1,211	$924	76.3%	4.33	2.81	38.66	14.78
TLogN	9.4	2.65	$1,297	$1,966	$669	51.6%	$1,264	-$33	-2.5%	$2,910	$1,192	41.0%	$2,832	$1,192	42.1%	$7,535	$4,089	54.3%	1.441	0.943	$1,481	$985	66.5%	6.55	3.47	61.51	20.09
TLogN	11	2.6	$4,208	$5,639	$1,431	34.0%	$4,337	$129	3.1%	$6,319	$4,072	64.4%	$6,154	$4,070	66.1%	$20,931	$14,366	68.6%	1.091	0.938	$3,951	$2,976	75.3%	4.86	3.98	36.00	24.79
TLogN	10	2.8	$4,145	$6,279	$2,134	51.5%	$4,177	$32	0.8%	$8,119	$3,972	48.9%	$7,833	$3,972	50.7%	$26,918	$15,066	56.0%	1.247	0.951	$5,076	$3,309	65.2%	4.54	2.87	29.89	11.67
	a	b																									
Logg	24	2.65	$192	$225	$33	17.0%	$199	$7	3.4%	$163	$137	84.1%	$159	$137	85.7%	$557	$478	85.9%	0.708	0.687	$155	$137	88.5%	3.06	2.94	17.56	16.31
Logg	33	3.3	$203	$225	$22	10.7%	$204	$1	0.4%	$131	$115	87.5%	$129	$115	88.7%	$492	$436	88.6%	0.575	0.562	$136	$121	88.9%	2.04	1.95	8.22	7.33
Logg	25	2.5	$1,064	$1,272	$208	19.5%	$1,105	$42	3.9%	$984	$814	82.7%	$962	$813	84.5%	$3,424	$2,849	83.2%	0.757	0.736	$936	$800	85.4%	2.56	2.47	10.93	10.19
Logg	34.5	3.15	$960	$1,090	$130	13.5%	$978	$18	1.8%	$734	$631	86.0%	$722	$631	87.3%	$2,505	$2,207	88.1%	0.663	0.645	$677	$594	87.7%	3.03	2.92	15.81	14.67
Logg	25.25	2.45	$1,877	$2,300	$423	22.5%	$1,986	$109	5.8%	$1,842	$1,507	81.8%	$1,793	$1,503	83.8%	$6,365	$5,300	83.3%	0.779	0.756	$1,762	$1,502	85.2%	3.01	2.87	18.19	16.53
Logg	34.7	3.07	$1,794	$2,097	$302	16.8%	$1,869	$74	4.1%	$1,500	$1,273	84.9%	$1,470	$1,271	86.5%	$5,113	$4,437	86.8%	0.701	0.680	$1,360	$1,203	88.4%	3.15	3.01	17.56	16.09
TLogg	23.5	2.65	$271	$496	$225	83.1%	$297	$27	9.9%	$903	$530	58.6%	$874	$529	60.5%	$2,359	$1,229	52.1%	1.764	1.778	$385	$217	56.4%	7.20	8.71	78.04	112.48
TLogg	33	3.3	$261	$382	$120	46.1%	$247	-$15	-5.6%	$458	$191	41.6%	$442	$190	42.9%	$1,462	$707	48.4%	1.159	0.771	$292	$184	62.8%	4.36	2.80	28.31	13.47
TLogg	24.5	2.5	$1,164	$2,152	$988	84.9%	$1,099	-$65	-5.6%	$3,620	$1,370	37.8%	$3,483	$1,368	39.3%	$11,139	$4,881	43.8%	1.618	1.245	$1,629	$782	48.0%	4.86	4.88	31.98	34.33
TLogg	34.5	3.15	$1,086	$1,437	$350	32.2%	$1,158	$72	6.6%	$1,453	$941	64.8%	$1,410	$938	66.5%	$4,792	$3,638	75.9%	0.981	0.810	$1,190	$935	78.5%	4.79	2.61	47.53	12.40
TLogg	24.75	2.45	$1,928	$3,618	$1,689	87.6%	$1,855	-$74	-3.8%	$6,604	$2,269	34.4%	$6,384	$2,267	35.5%	$20,825	$6,298	30.2%	1.765	1.223	$2,710	$1,406	51.9%	6.84	6.34	72.80	61.09
TLogg	34.6	3.07	$1,892	$2,493	$601	31.8%	$2,050	$158	8.4%	$2,499	$1,764	70.6%	$2,425	$1,757	72.4%	$8,627	$6,151	71.3%	0.973	0.857	$1,861	$1,517	81.5%	3.59	2.90	21.03	13.68
	ξ	θ																									
GPD	0.8	35,000	$382	$696	$313	81.9%	$396	$13	3.5%	$1,069	$521	48.7%	$1,022	$521	50.9%	$3,387	$1,746	51.5%	1.469	1.315	$577	$332	57.5%	4.38	3.97	27.86	23.37
GPD	0.95	7,500	$375	$785	$410	109.2%	$390	$14	3.8%	$1,398	$588	42.1%	$1,336	$588	44.0%	$4,041	$1,906	47.2%	1.701	1.509	$657	$330	50.2%	5.16	4.69	39.16	33.55
GPD	0.875	47,500	$1,106	$2,123	$1,016	91.9%	$1,130	$24	2.2%	$3,514	$1,594	45.4%	$3,363	$1,594	47.4%	$10,348	$5,113	49.4%	1.585	1.410	$1,818	$960	52.8%	4.87	4.41	34.76	29.38
GPD	0.95	25,000	$1,251	$2,576	$1,325	105.9%	$1,279	$28	2.2%	$4,585	$1,930	42.1%	$4,390	$1,930	44.0%	$13,301	$6,281	47.2%	1.704	1.509	$2,182	$1,099	50.4%	5.30	4.81	41.36	35.48
GPD	0.925	50,000	$1,938	$3,835	$1,898	97.9%	$1,955	$17	0.9%	$6,657	$2,882	43.3%	$6,381	$2,882	45.2%	$19,608	$9,118	46.5%	1.664	1.475	$3,298	$1,681	51.0%	5.28	4.76	41.18	34.90
GPD	0.99	27,500	$2,076	$4,375	$2,299	110.7%	$2,095	$19	0.9%	$8,085	$3,275	40.5%	$7,751	$3,275	42.2%	$23,030	$10,486	45.5%	1.772	1.563	$3,686	$1,797	48.8%	5.68	5.15	47.71	41.14
TGPD	0.775	33,500	$351	$617	$266	75.7%	$365	$13	3.8%	$1,109	$543	48.9%	$1,077	$543	50.4%	$2,654	$1,446	54.5%	1.745	1.489	$515	$310	60.2%	9.98	8.34	164.33	121.42
TGPD	0.8	25,000	$361	$660	$299	82.9%	$379	$18	5.0%	$1,193	$579	48.5%	$1,155	$578	50.1%	$3,435	$1,776	51.7%	1.749	1.526	$525	$307	58.6%	7.36	6.79	81.65	72.55
TGPD	0.8675	50,000	$1,267	$2,432	$1,166	92.0%	$1,327	$61	4.8%	$4,337	$1,988	45.9%	$4,177	$1,988	47.6%	$12,419	$6,337	51.0%	1.717	1.498	$2,065	$1,144	55.4%	5.54	4.89	43.68	35.05
TGPD	0.91	31,000	$1,334	$2,672	$1,338	100.4%	$1,389	$55	4.1%	$5,165	$2,203	42.6%	$4,989	$2,202	44.1%	$13,302	$6,344	47.7%	1.867	1.586	$2,224	$1,182	53.2%	6.58	5.48	63.72	45.31
TGPD	0.92	47,500	$2,088	$4,048	$1,960	93.9%	$2,129	$41	2.0%	$7,203	$3,235	44.9%	$6,931	$3,235	46.7%	$20,816	$10,223	49.1%	1.712	1.519	$3,384	$1,765	52.2%	5.13	4.60	36.26	28.88
TGPD	0.95	35,000	$2,227	$4,474	$2,246	100.8%	$2,246	$19	0.8%	$9,606	$3,873	40.3%	$9,339	$3,873	41.5%	$22,320	$10,320	46.2%	2.088	1.724	$3,650	$1,936	53.1%	9.86	7.85	155.89	104.71

*NOTE: #simulations = 1,000; λ = 25 for 10 years so n ~ 250; α=0.9997

TABLE F5a
RCE vs. LDA for Regulatory Capital Estimation Under iid ($m, λ=15)*

Severity Dist.	Parm1	Parm2	True RCap	Mean MLE RCap	MLE Bias	MLE Bias%	Mean RCE RCap	RCE Bias	RCE Bias%	RMSE MLE RCap	RMSE RCE RCap	RMSE RCap RCE/MLE	StdDev MLE RCap	StdDev RCE RCap	StdDev RCap RCE/MLE	95%CIs MLE RCap	95%CIs RCE RCap	95%CIs RCap RCE/MLE	CV MLE	CV RCE	IQR MLE RCap	IQR RCE RCap	IQR RCap RCE/MLE	Skew MLE RCap	Skew RCE RCap	Kurtosis MLE RCap	Kurtosis RCE RCap
	μ	σ																									
LogN	10	2	$48	$51	$3	6.0%	$47	-$1	-1.6%	$24	$22	90.9%	$24	$22	91.5%	$92	$85	91.8%	0.468	0.461	$29	$26	91.0%	1.20	1.19	1.86	1.81
LogN	7.7	2.55	$38	$43	$5	11.9%	$38	$0	-0.2%	$27	$23	86.3%	$27	$23	87.6%	$101	$90	88.3%	0.617	0.606	$30	$26	87.4%	1.59	1.57	3.44	3.33
LogN	10.4	2.5	$473	$527	$54	11.3%	$471	-$2	-0.4%	$322	$279	86.8%	$318	$279	88.0%	$1,216	$1,078	88.7%	0.603	0.593	$358	$314	87.7%	1.55	1.53	3.27	3.17
LogN	9.27	2.77	$426	$489	$63	14.9%	$428	$2	0.5%	$337	$285	84.4%	$331	$285	85.9%	$1,262	$1,093	86.6%	0.678	0.665	$359	$308	85.8%	1.75	1.72	4.22	4.08
LogN	10.75	2.7	$1,434	$1,633	$199	13.9%	$1,438	$4	0.3%	$1,093	$929	85.0%	$1,075	$929	86.4%	$4,097	$3,573	87.2%	0.658	0.646	$1,177	$1,015	86.3%	1.70	1.67	3.96	3.83
LogN	9.63	2.97	$1,306	$1,540	$233	17.9%	$1,324	$17	1.3%	$1,154	$952	82.5%	$1,130	$952	84.2%	$4,283	$3,641	85.0%	0.734	0.719	$1,184	$997	84.3%	1.90	1.86	5.03	4.84
TLogN	10.2	1.95	$59	$70	$11	18.9%	$59	$0	0.3%	$55	$39	71.4%	$54	$39	72.9%	$185	$143	77.5%	0.771	0.667	$49	$40	81.0%	3.63	2.65	24.46	13.60
TLogN	9	2.2	$57	$87	$30	52.0%	$56	-$1	-2.1%	$173	$44	25.4%	$171	$44	25.8%	$350	$169	48.3%	1.955	0.782	$64	$45	70.0%	17.77	2.05	433.88	5.66
TLogN	10.7	2.385	$497	$710	$213	42.9%	$535	$39	7.8%	$1,072	$537	50.1%	$1,050	$535	51.0%	$2,468	$1,661	67.3%	1.479	1.000	$505	$396	78.5%	9.59	4.76	133.49	36.93
TLogN	9.4	2.65	$465	$764	$299	64.2%	$457	-$8	-1.8%	$1,116	$434	38.9%	$1,075	$434	40.4%	$3,540	$1,699	48.0%	1.408	0.950	$611	$386	63.2%	4.75	3.00	32.25	14.29
TLogN	11	2.6	$1,510	$2,263	$753	49.9%	$1,677	$167	11.1%	$2,740	$1,676	61.2%	$2,635	$1,668	63.3%	$9,381	$5,847	62.3%	1.164	0.994	$1,754	$1,331	75.9%	3.77	3.38	18.98	16.96
TLogN	10	2.8	$1,389	$2,652	$1,263	90.9%	$1,507	$118	8.5%	$5,725	$1,743	30.4%	$5,584	$1,739	31.1%	$12,350	$5,777	46.8%	2.105	1.154	$1,924	$1,239	64.4%	10.36	3.82	167.66	21.93
	a	b																									
Logg	24	2.65	$59	$73	$13	22.3%	$60	$1	1.2%	$60	$46	76.6%	$58	$46	78.5%	$217	$173	79.9%	0.802	0.761	$52	$42	81.3%	2.86	2.68	12.85	11.31
Logg	33	3.3	$72	$85	$13	17.8%	$73	$1	1.3%	$61	$49	80.6%	$60	$49	82.4%	$210	$174	83.1%	0.707	0.677	$59	$50	84.4%	2.69	2.57	12.04	11.24
Logg	25	2.5	$301	$380	$79	26.4%	$306	$5	1.7%	$344	$253	73.4%	$335	$253	75.4%	$1,142	$863	75.6%	0.881	0.826	$277	$221	79.8%	3.39	3.07	18.59	15.01
Logg	34.5	3.15	$317	$369	$52	16.3%	$312	-$5	-1.6%	$262	$209	79.9%	$257	$209	81.5%	$981	$796	81.1%	0.697	0.670	$270	$224	82.9%	2.06	2.00	5.94	5.66
Logg	25.25	2.45	$515	$659	$144	27.9%	$523	$8	1.6%	$607	$440	72.6%	$589	$440	74.7%	$2,135	$1,555	72.8%	0.894	0.841	$490	$384	78.3%	3.04	2.86	14.03	12.53
Logg	34.7	3.07	$575	$685	$111	19.3%	$574	$0	-0.1%	$522	$411	78.6%	$511	$411	80.5%	$1,806	$1,463	81.0%	0.745	0.716	$523	$431	82.3%	2.34	2.23	9.29	8.39
TLogg	23.5	2.65	$87	$232	$145	166.7%	$150	$63	72.8%	$1,121	$372	33.2%	$1,112	$366	33.0%	$1,156	$928	80.3%	4.789	2.436	$145	$98	67.3%	24.42	8.44	688.38	95.43
TLogg	33	3.3	$94	$162	$68	72.7%	$108	$14	15.4%	$398	$251	62.9%	$392	$250	63.8%	$726	$399	55.0%	2.422	2.311	$110	$71	64.7%	19.94	22.43	514.19	616.69
TLogg	24.5	2.5	$339	$712	$373	110.2%	$408	$70	20.6%	$1,547	$825	53.3%	$1,502	$822	54.7%	$3,703	$1,829	49.4%	2.109	2.014	$522	$282	54.0%	7.01	7.66	63.01	78.35
TLogg	34.5	3.15	$362	$526	$164	45.3%	$380	$18	5.0%	$704	$335	47.5%	$685	$334	48.8%	$2,027	$1,189	58.6%	1.302	0.879	$417	$304	72.8%	5.55	3.05	48.31	16.46
TLogg	24.75	2.45	$543	$1,097	$554	102.1%	$586	$43	8.0%	$2,098	$999	47.6%	$2,023	$998	49.3%	$6,613	$2,989	45.2%	1.845	1.703	$802	$415	51.7%	5.41	5.80	39.73	47.30
TLogg	34.6	3.07	$610	$858	$248	40.7%	$668	$58	9.5%	$962	$576	59.9%	$929	$573	61.7%	$3,157	$2,027	64.2%	1.083	0.858	$725	$577	79.6%	3.67	2.39	22.49	9.30
	ξ	θ																									
GPD	0.8	35,000	$99	$178	$79	80.3%	$101	$2	2.4%	$257	$123	47.6%	$245	$123	50.0%	$868	$450	51.8%	1.378	1.215	$165	$94	57.0%	3.30	2.89	14.01	10.60
GPD	0.95	7,500	$74	$155	$81	108.8%	$76	$2	2.6%	$255	$103	40.6%	$242	$103	42.8%	$837	$368	44.0%	1.558	1.356	$139	$70	50.5%	3.68	3.13	17.78	12.60
GPD	0.875	47,500	$250	$477	$227	91.1%	$253	$4	1.4%	$725	$320	44.2%	$688	$320	46.5%	$2,444	$1,176	48.1%	1.443	1.265	$443	$237	53.6%	3.47	2.97	15.94	11.37
GPD	0.95	25,000	$248	$509	$262	105.7%	$251	$3	1.4%	$832	$339	40.7%	$790	$339	42.9%	$2,694	$1,226	45.5%	1.551	1.350	$458	$233	50.9%	3.68	3.13	18.11	12.67
GPD	0.925	50,000	$401	$761	$360	90.0%	$386	-$14	-3.6%	$1,245	$528	42.4%	$1,192	$527	44.2%	$4,185	$1,879	44.9%	1.566	1.366	$703	$365	51.9%	3.70	3.14	18.55	12.95
GPD	0.99	27,500	$383	$802	$419	109.5%	$381	-$2	-0.6%	$1,348	$527	39.1%	$1,281	$527	41.2%	$4,375	$1,901	43.5%	1.596	1.385	$722	$355	49.2%	3.90	3.28	20.94	14.37
TGPD	0.775	33,500	$95	$172	$77	81.6%	$100	$5	5.4%	$323	$157	48.6%	$313	$157	50.0%	$909	$451	49.6%	1.824	1.572	$148	$85	57.5%	6.48	5.97	57.49	50.13
TGPD	0.8	25,000	$93	$159	$66	71.3%	$92	-$1	-0.8%	$250	$122	48.8%	$241	$122	50.6%	$748	$393	52.6%	1.513	1.322	$149	$84	56.6%	5.16	4.59	41.59	34.13
TGPD	0.8675	50,000	$290	$583	$293	101.2%	$315	$26	8.8%	$964	$427	44.3%	$918	$426	46.4%	$3,225	$1,570	48.7%	1.574	1.351	$527	$283	53.7%	4.74	4.00	31.27	22.49
TGPD	0.91	31,000	$283	$549	$266	93.8%	$283	-$1	-0.2%	$1,053	$438	41.6%	$1,019	$438	43.0%	$2,953	$1,375	46.6%	1.857	1.552	$467	$250	53.5%	7.01	5.73	81.71	56.83
TGPD	0.92	47,500	$436	$940	$505	115.9%	$483	$47	10.8%	$1,567	$657	41.9%	$1,483	$655	44.2%	$5,210	$2,361	45.3%	1.578	1.358	$912	$480	52.6%	4.18	3.60	22.83	17.62
TGPD	0.95	35,000	$441	$906	$465	105.6%	$446	$5	1.2%	$2,059	$801	38.9%	$2,005	$801	39.9%	$4,830	$2,241	46.4%	2.213	1.795	$710	$365	51.5%	7.64	6.32	76.96	54.31

*NOTE: #simulations = 1,000; λ = 15 for 10 years so n ~ 150; α=0.999

TABLE F5b

RCE vs. LDA for Economic Capital Estimation Under iid ($m, λ=15)*

Severity Dist.	Parm1	Parm2	True ECap	Mean MLE ECap	MLE Bias	MLE Bias%	Mean RCE ECap	RCE Bias	RCE Bias%	RMSE MLE ECap	RMSE RCE ECap	RMSE ECap RCE/MLE	StdDev MLE ECap	StdDev RCE ECap	StdDev ECap RCE/MLE	95%CIs MLE ECap	95%CIs RCE ECap	95%CIs ECap RCE/MLE	CV MLE	CV RCE	IQR MLE ECap	IQR RCE ECap	IQR ECap RCE/MLE	Skew MLE ECap	Skew RCE ECap	Kurtosis MLE ECap	Kurtosis RCE ECap
	μ	σ																									
LogN	10	2	$84	$90	$6	7.3%	$83	-$1	-0.8%	$46	$41	90.4%	$45	$41	91.2%	$179	$162	90.5%	0.504	0.497	$54	$49	90.9%	1.29	1.28	2.16	2.10
LogN	7.7	2.55	$79	$90	$11	14.2%	$80	$1	1.2%	$61	$52	85.6%	$60	$52	87.1%	$235	$202	86.1%	0.663	0.651	$66	$57	87.3%	1.71	1.67	3.94	3.80
LogN	10.4	2.5	$958	$1,087	$129	13.5%	$968	$10	1.0%	$717	$617	86.1%	$705	$617	87.5%	$2,772	$2,399	86.6%	0.649	0.637	$781	$685	87.7%	1.67	1.64	3.75	3.62
LogN	9.27	2.77	$935	$1,099	$165	17.6%	$955	$21	2.2%	$818	$683	83.5%	$801	$683	85.2%	$3,140	$2,642	84.1%	0.729	0.714	$846	$723	85.5%	1.88	1.84	4.83	4.64
LogN	10.75	2.7	$3,082	$3,590	$508	16.5%	$3,141	$58	1.9%	$2,591	$2,181	84.2%	$2,541	$2,180	85.8%	$9,968	$8,451	84.8%	0.708	0.694	$2,718	$2,339	86.1%	1.82	1.79	4.54	4.36
LogN	9.63	2.97	$3,041	$3,682	$641	21.1%	$3,140	$99	3.2%	$2,977	$2,428	81.6%	$2,907	$2,426	83.5%	$11,363	$9,348	82.3%	0.789	0.773	$2,955	$2,474	83.7%	2.04	1.99	5.74	5.49
TLogN	10.2	1.95	$99	$124	$24	24.6%	$101	$2	1.7%	$115	$76	66.4%	$113	$76	67.9%	$362	$279	77.1%	0.911	0.758	$92	$73	79.3%	4.51	3.13	35.70	18.58
TLogN	9	2.2	$103	$182	$79	76.8%	$101	-$2	-2.0%	$543	$88	16.2%	$537	$88	16.3%	$805	$332	41.2%	2.951	0.869	$128	$85	66.6%	22.07	2.27	592.72	7.09
TLogN	10.7	2.385	$959	$1,509	$549	57.2%	$1,062	$103	10.7%	$2,960	$1,225	41.4%	$2,908	$1,220	42.0%	$5,611	$3,713	66.2%	1.928	1.149	$1,075	$801	74.5%	12.09	5.47	200.76	46.47
TLogN	9.4	2.65	$959	$1,801	$842	87.8%	$943	-$16	-1.7%	$3,173	$994	31.3%	$3,059	$994	32.5%	$9,115	$3,895	42.7%	1.699	1.054	$1,407	$824	58.5%	5.58	3.37	42.63	18.43
TLogN	11	2.6	$3,113	$5,093	$1,980	63.6%	$3,551	$438	14.1%	$7,129	$3,969	55.7%	$6,848	$3,945	57.6%	$23,464	$13,431	57.2%	1.345	1.111	$3,941	$2,864	72.7%	4.16	3.66	22.33	19.17
TLogN	10	2.8	$3,009	$6,838	$3,829	127.3%	$3,346	$338	11.2%	$18,723	$4,465	23.8%	$18,327	$4,453	24.3%	$33,900	$13,664	40.3%	2.680	1.331	$4,582	$2,803	61.2%	12.27	4.65	220.68	32.36
	a	b																									
Logg	24	2.65	$136	$174	$38	28.3%	$139	$4	2.6%	$164	$120	73.1%	$159	$120	75.1%	$581	$438	75.4%	0.915	0.859	$129	$102	78.7%	3.31	3.06	17.04	14.65
Logg	33	3.3	$149	$182	$33	22.2%	$153	$4	2.6%	$149	$116	78.0%	$145	$116	79.9%	$502	$406	80.9%	0.798	0.760	$134	$112	83.7%	3.02	2.87	14.80	13.64
Logg	25	2.5	$732	$976	$244	33.3%	$754	$22	3.0%	$1,016	$705	69.4%	$987	$705	71.4%	$3,274	$2,368	72.3%	1.011	0.934	$747	$567	75.9%	3.99	3.51	25.88	19.49
Logg	34.5	3.15	$691	$833	$142	20.5%	$686	-$5	-0.7%	$663	$510	77.0%	$647	$510	78.9%	$2,401	$1,942	80.9%	0.777	0.744	$633	$517	81.7%	2.26	2.18	7.08	6.65
Logg	25.25	2.45	$1,281	$1,733	$452	35.2%	$1,319	$38	3.0%	$1,811	$1,244	68.7%	$1,753	$1,244	70.9%	$6,125	$4,363	71.2%	1.012	0.943	$1,349	$1,016	75.3%	3.36	3.14	16.76	14.89
Logg	34.7	3.07	$1,280	$1,589	$309	24.2%	$1,294	$14	1.1%	$1,357	$1,027	75.7%	$1,321	$1,027	77.7%	$4,567	$3,618	79.2%	0.831	0.794	$1,279	$1,020	79.7%	2.60	2.47	11.36	10.18
TLogg	23.5	2.65	$193	$828	$635	329.3%	$416	$223	115.7%	$7,242	$1,460	20.2%	$7,214	$1,443	20.0%	$3,903	$2,804	71.8%	8.715	3.470	$382	$213	55.7%	28.55	11.01	865.50	159.83
TLogg	33	3.3	$192	$404	$212	110.5%	$220	$29	14.9%	$1,465	$795	54.3%	$1,450	$795	54.8%	$2,039	$897	44.0%	3.588	3.605	$257	$143	55.8%	23.68	26.41	659.71	777.87
TLogg	24.5	2.5	$807	$2,175	$1,368	169.5%	$985	$178	22.0%	$6,125	$2,780	45.4%	$5,970	$2,774	46.5%	$13,076	$4,519	34.6%	2.745	2.816	$1,423	$632	44.5%	8.45	10.62	89.14	150.22
TLogg	34.5	3.15	$783	$1,278	$495	63.3%	$789	$6	0.8%	$2,135	$759	35.6%	$2,077	$759	36.5%	$5,585	$2,599	46.5%	1.625	0.962	$1,015	$642	63.2%	6.84	3.75	70.02	26.23
TLogg	24.75	2.45	$1,325	$3,342	$2,017	152.3%	$1,357	$32	2.4%	$7,938	$2,981	37.6%	$7,677	$2,981	38.8%	$22,129	$7,350	33.2%	2.297	2.197	$2,278	$905	39.7%	6.68	7.58	63.01	82.42
TLogg	34.6	3.07	$1,352	$2,095	$744	55.0%	$1,458	$106	7.8%	$2,805	$1,352	48.2%	$2,704	$1,348	49.8%	$8,667	$4,701	54.2%	1.291	0.925	$1,797	$1,288	71.7%	4.65	2.62	36.54	11.51
	ξ	θ																									
GPD	0.8	35,000	$254	$558	$304	119.9%	$263	$9	3.4%	$976	$378	38.7%	$928	$378	40.7%	$3,208	$1,351	42.1%	1.661	1.438	$494	$244	49.5%	3.89	3.35	19.39	14.17
GPD	0.95	7,500	$231	$608	$377	163.4%	$238	$7	3.1%	$1,199	$380	31.7%	$1,138	$380	33.4%	$3,745	$1,325	35.4%	1.871	1.598	$515	$215	41.8%	4.35	3.64	24.85	16.90
GPD	0.875	47,500	$707	$1,668	$961	135.9%	$720	$13	1.9%	$3,053	$1,075	35.2%	$2,898	$1,074	37.1%	$9,646	$3,778	39.2%	1.737	1.492	$1,479	$678	45.8%	4.14	3.47	22.76	15.63
GPD	0.95	25,000	$770	$1,992	$1,222	158.8%	$782	$12	1.5%	$3,909	$1,243	31.8%	$3,713	$1,243	33.5%	$12,270	$4,393	35.8%	1.863	1.590	$1,700	$705	41.5%	4.38	3.64	25.62	17.12
GPD	0.925	50,000	$1,207	$2,869	$1,661	137.6%	$1,167	-$41	-3.4%	$5,633	$1,874	33.3%	$5,382	$1,873	34.8%	$17,924	$6,552	36.6%	1.876	1.606	$2,498	$1,066	42.7%	4.44	3.68	26.65	17.79
GPD	0.99	27,500	$1,252	$3,315	$2,063	164.9%	$1,241	-$11	-0.9%	$6,693	$2,024	30.2%	$6,367	$2,024	31.8%	$20,958	$6,920	33.0%	1.921	1.631	$2,784	$1,120	40.2%	4.68	3.82	29.99	19.38
TGPD	0.775	33,500	$236	$538	$302	127.8%	$257	$21	9.0%	$1,295	$508	39.3%	$1,259	$508	40.3%	$3,192	$1,326	41.5%	2.341	1.974	$443	$216	48.7%	7.68	7.08	77.92	67.73
TGPD	0.8	25,000	$240	$499	$260	108.5%	$239	$0	0.0%	$985	$390	39.6%	$950	$390	41.0%	$2,696	$1,162	43.1%	1.902	1.627	$452	$218	48.1%	6.53	5.83	63.93	53.24
TGPD	0.8675	50,000	$813	$2,071	$1,258	154.8%	$910	$97	12.0%	$4,252	$1,482	34.9%	$4,061	$1,479	36.4%	$12,752	$5,320	41.7%	1.961	1.625	$1,770	$818	46.2%	5.72	4.70	44.43	30.37
TGPD	0.91	31,000	$837	$2,065	$1,228	146.7%	$846	$9	1.0%	$5,096	$1,638	32.1%	$4,945	$1,638	33.1%	$12,708	$4,708	37.1%	2.394	1.936	$1,629	$727	44.6%	9.16	7.50	132.86	94.59
TGPD	0.92	47,500	$1,304	$3,609	$2,304	176.7%	$1,483	$179	13.7%	$7,331	$2,401	32.7%	$6,959	$2,394	34.4%	$22,760	$8,367	36.8%	1.928	1.614	$3,330	$1,464	44.0%	4.93	4.22	30.61	23.44
TGPD	0.95	35,000	$1,371	$3,681	$2,310	168.6%	$1,416	$45	3.3%	$10,884	$3,185	29.3%	$10,636	$3,184	29.9%	$21,357	$8,114	38.0%	2.889	2.249	$2,580	$1,100	42.6%	9.09	7.37	104.00	70.72

*NOTE: #simulations = 1,000; λ = 15 for 10 years so n ~ 150; α=0.9997

TABLE F6a

RCE vs. LDA for Regulatory Capital Estimation Under iid ($m, λ=50)*

Severity Dist.	Parm1	Parm2	True RCap	Mean MLE RCap	MLE Bias	MLE Bias%	Mean RCE RCap	RCE Bias	RCE Bias%	RMSE MLE RCap	RMSE RCE RCap	RMSE RCap RCE/MLE	StdDev MLE RCap	StdDev RCE RCap	StdDev RCap RCE/MLE	95%CIs MLE RCap	95%CIs RCE RCap	95%CIs RCap RCE/MLE	CV MLE	CV RCE	IQR MLE RCap	IQR RCE RCap	IQR RCap RCE/MLE	Skew MLE RCap	Skew RCE RCap	Kurtosis MLE RCap	Kurtosis RCE RCap
	μ	σ																									
LogN	10	2	$90	$92	$3	3.0%	$89	$0	-0.2%	$25	$24	95.7%	$25	$24	96.3%	$97	$93	96.0%	0.272	0.270	$31	$30	95.9%	0.86	0.85	0.90	0.88
LogN	7.7	2.55	$81	$85	$4	5.4%	$81	$0	0.3%	$31	$29	93.5%	$31	$29	94.4%	$118	$112	94.5%	0.362	0.359	$36	$34	95.5%	1.10	1.08	1.52	1.49
LogN	10.4	2.5	$984	$1,035	$51	5.1%	$987	$3	0.3%	$370	$347	93.7%	$367	$347	94.6%	$1,402	$1,327	94.7%	0.354	0.351	$429	$411	95.8%	1.07	1.06	1.46	1.43
LogN	9.27	2.77	$952	$1,014	$62	6.5%	$958	$6	0.6%	$408	$377	92.5%	$403	$377	93.6%	$1,534	$1,437	93.7%	0.397	0.394	$462	$438	94.8%	1.19	1.18	1.82	1.78
LogN	10.75	2.7	$3,145	$3,338	$194	6.2%	$3,161	$16	0.5%	$1,304	$1,210	92.8%	$1,289	$1,210	93.8%	$4,914	$4,617	94.0%	0.386	0.383	$1,487	$1,414	95.1%	1.16	1.15	1.72	1.69
LogN	9.63	2.97	$3,085	$3,321	$236	7.7%	$3,112	$27	0.9%	$1,446	$1,324	91.6%	$1,427	$1,324	92.8%	$5,412	$5,027	92.9%	0.430	0.425	$1,612	$1,515	94.0%	1.28	1.26	2.10	2.06
TLogN	10.2	1.95	$108	$117	$9	8.1%	$109	$1	1.0%	$49	$44	88.3%	$49	$44	89.6%	$192	$175	90.7%	0.416	0.399	$54	$49	92.3%	1.64	1.54	4.25	3.70
TLogN	9	2.2	$110	$125	$15	13.9%	$111	$1	1.3%	$76	$61	80.0%	$75	$61	81.6%	$272	$224	82.6%	0.596	0.547	$74	$63	86.0%	2.71	2.35	14.19	10.59
TLogN	10.7	2.385	$996	$1,120	$124	12.5%	$1,014	$18	1.8%	$620	$527	84.9%	$608	$526	86.6%	$2,192	$1,932	88.1%	0.543	0.519	$615	$547	88.9%	2.06	1.97	7.57	6.98
TLogN	9.4	2.65	$985	$1,183	$197	20.0%	$1,006	$21	2.1%	$838	$637	76.0%	$815	$637	78.1%	$3,008	$2,384	79.3%	0.689	0.633	$799	$663	82.9%	2.56	2.18	12.21	8.07
TLogN	11	2.6	$3,195	$3,667	$472	14.8%	$3,270	$75	2.4%	$2,201	$1,834	83.3%	$2,150	$1,832	85.2%	$8,260	$6,980	84.5%	0.586	0.560	$2,178	$1,910	87.7%	2.29	2.15	9.06	8.02
TLogN	10	2.8	$3,072	$3,599	$527	17.1%	$3,061	-$11	-0.4%	$2,422	$1,864	77.0%	$2,364	$1,864	78.9%	$8,573	$6,851	79.9%	0.657	0.609	$2,401	$1,984	82.6%	2.46	2.18	11.43	9.04
	a	b																									
Logg	24	2.65	$139	$146	$8	5.6%	$139	$0	0.0%	$64	$60	93.0%	$64	$60	93.7%	$246	$233	94.5%	0.437	0.432	$73	$68	93.1%	1.65	1.63	4.61	4.47
Logg	33	3.3	$154	$160	$6	3.6%	$153	-$1	-0.8%	$58	$55	94.4%	$58	$55	94.8%	$223	$213	95.3%	0.363	0.360	$71	$68	95.8%	1.26	1.24	2.45	2.36
Logg	25	2.5	$745	$806	$62	8.3%	$758	$13	1.8%	$382	$351	91.7%	$377	$350	92.8%	$1,423	$1,327	93.2%	0.468	0.462	$433	$406	93.6%	1.51	1.48	3.75	3.61
Logg	34.5	3.15	$709	$754	$45	6.3%	$718	$9	1.2%	$318	$297	93.4%	$315	$297	94.3%	$1,188	$1,118	94.1%	0.417	0.413	$372	$349	93.8%	1.29	1.28	2.55	2.50
Logg	25.25	2.45	$1,301	$1,425	$124	9.5%	$1,337	$36	2.7%	$710	$649	91.5%	$699	$648	92.8%	$2,698	$2,517	93.3%	0.490	0.485	$807	$754	93.4%	1.51	1.50	3.56	3.49
Logg	34.7	3.07	$1,310	$1,403	$93	7.1%	$1,333	$23	1.7%	$611	$569	93.1%	$604	$569	94.1%	$2,194	$2,061	94.0%	0.430	0.426	$710	$674	94.8%	1.69	1.67	6.22	6.05
TLogg	23.5	2.65	$199	$262	$63	31.7%	$204	$5	2.5%	$310	$182	58.7%	$304	$182	59.9%	$808	$547	67.7%	1.161	0.894	$182	$136	74.8%	8.04	5.32	110.67	49.66
TLogg	33	3.3	$200	$226	$26	13.2%	$204	$4	2.0%	$155	$124	80.4%	$152	$124	81.6%	$503	$439	87.3%	0.674	0.610	$130	$116	88.9%	5.56	4.31	71.10	45.45
TLogg	24.5	2.5	$823	$1,035	$212	25.8%	$860	$37	4.5%	$901	$634	70.3%	$876	$633	72.3%	$3,051	$2,236	73.3%	0.846	0.736	$719	$590	82.0%	3.53	2.77	20.37	12.59
TLogg	34.5	3.15	$805	$899	$93	11.6%	$827	$21	2.6%	$510	$446	87.5%	$502	$446	88.9%	$1,941	$1,745	89.9%	0.558	0.540	$572	$507	88.7%	1.56	1.48	3.36	2.96
TLogg	24.75	2.45	$1,348	$1,664	$316	23.4%	$1,400	$52	3.9%	$1,455	$1,048	72.0%	$1,421	$1,047	73.7%	$4,562	$3,529	77.4%	0.854	0.748	$1,238	$1,005	81.1%	4.21	3.10	32.51	17.37
TLogg	34.6	3.07	$1,386	$1,573	$188	13.6%	$1,450	$65	4.7%	$890	$778	87.4%	$870	$775	89.1%	$3,272	$2,914	89.1%	0.553	0.535	$973	$875	89.9%	2.11	1.97	9.53	8.36
	ξ	θ																									
GPD	0.8	35,000	$260	$339	$79	30.5%	$268	$8	3.0%	$304	$220	72.4%	$294	$220	74.9%	$1,054	$792	75.2%	0.866	0.822	$276	$213	77.1%	2.90	2.73	12.83	11.32
GPD	0.95	7,500	$234	$327	$93	39.8%	$243	$10	4.1%	$339	$230	67.8%	$326	$230	70.4%	$1,154	$827	71.6%	0.998	0.944	$276	$204	74.0%	3.33	3.10	16.90	14.51
GPD	0.875	47,500	$719	$970	$251	34.8%	$744	$25	3.4%	$939	$658	70.1%	$905	$658	72.7%	$3,219	$2,368	73.6%	0.933	0.885	$808	$600	74.2%	3.13	2.94	14.95	13.16
GPD	0.95	25,000	$779	$1,088	$309	39.7%	$810	$31	4.0%	$1,129	$765	67.8%	$1,086	$765	70.4%	$3,847	$2,757	71.6%	0.998	0.944	$918	$681	74.2%	3.34	3.10	16.97	14.56
GPD	0.925	50,000	$1,223	$1,681	$458	37.4%	$1,265	$41	3.4%	$1,701	$1,168	68.7%	$1,638	$1,167	71.3%	$5,647	$4,037	71.5%	0.974	0.923	$1,411	$1,040	73.7%	3.30	3.08	16.65	14.48
GPD	0.99	27,500	$1,264	$1,790	$526	41.6%	$1,311	$48	3.8%	$1,917	$1,275	66.5%	$1,843	$1,274	69.1%	$6,396	$4,424	69.2%	1.030	0.971	$1,530	$1,119	73.1%	3.49	3.22	18.80	15.93
TGPD	0.775	33,500	$243	$304	$61	25.3%	$243	$0	-0.1%	$249	$183	73.5%	$242	$183	75.9%	$976	$741	75.9%	0.794	0.755	$224	$177	79.2%	2.42	2.30	8.75	7.88
TGPD	0.8	25,000	$246	$315	$70	28.3%	$249	$3	1.2%	$282	$205	72.6%	$273	$205	74.9%	$984	$745	75.7%	0.867	0.823	$244	$189	77.7%	2.92	2.77	13.13	11.89
TGPD	0.8675	50,000	$827	$1,101	$274	33.1%	$845	$18	2.2%	$1,104	$771	69.8%	$1,069	$770	72.1%	$3,440	$2,530	73.6%	0.971	0.912	$868	$647	74.5%	4.07	3.72	28.67	24.11
TGPD	0.91	31,000	$850	$1,140	$290	34.1%	$861	$11	1.3%	$1,091	$750	68.7%	$1,051	$750	71.3%	$3,764	$2,690	71.5%	0.922	0.870	$915	$672	73.5%	3.53	3.19	25.80	21.07
TGPD	0.92	47,500	$1,323	$1,795	$472	35.7%	$1,351	$28	2.1%	$1,800	$1,240	68.9%	$1,736	$1,240	71.4%	$6,365	$4,514	70.9%	0.967	0.918	$1,481	$1,095	73.9%	3.31	3.13	18.39	16.56
TGPD	0.95	35,000	$1,387	$1,928	$541	39.0%	$1,428	$41	2.9%	$2,245	$1,487	66.2%	$2,179	$1,486	68.2%	$6,423	$4,549	70.8%	1.130	1.041	$1,592	$1,168	73.3%	6.65	5.70	83.48	63.04

*NOTE: #simulations = 1,000; λ = 50 for 10 years so n ~ 500; α=0.999

TABLE F6b
RCE vs. LDA for Economic Capital Estimation Under iid ($m, λ=50)*

Severity Dist.	Parm1	Parm2	True ECap	Mean MLE ECap	MLE Bias	MLE Bias%	Mean RCE ECap	RCE Bias	RCE Bias%	RMSE MLE ECap	RMSE RCE ECap	RMSE ECap RCE/MLE	StdDev MLE ECap	StdDev RCE ECap	StdDev ECap RCE/MLE	95%CIs MLE ECap	95%CIs RCE ECap	95%CIs ECap RCE/MLE	CV MLE	CV RCE	IQR MLE ECap	IQR RCE ECap	IQR ECap RCE/MLE	Skew MLE ECap	Skew RCE ECap	Kurtosis MLE ECap	Kurtosis RCE ECap
	μ	**σ**																									
LogN	10	2	$148	$153	**$5**	**3.5%**	$148	**$0**	**0.1%**	$45	$43	**95.5%**	$45	$43	96.1%	$173	$166	**95.8%**	0.293	0.291	$55	$53	96.6%	0.91	0.90	1.02	1.01
LogN	7.7	2.55	$158	$168	**$10**	**6.2%**	$160	**$1**	**0.8%**	$66	$61	**93.1%**	$65	$61	94.1%	$249	$234	**94.0%**	0.387	0.384	$75	$72	94.8%	1.16	1.15	1.72	1.68
LogN	10.4	2.5	$1,898	$2,010	**$112**	**5.9%**	$1,912	**$15**	**0.8%**	$769	$718	**93.3%**	$761	$718	94.3%	$2,917	$2,747	**94.2%**	0.379	0.375	$886	$841	94.9%	1.14	1.13	1.65	1.61
LogN	9.27	2.77	$1,984	$2,133	**$149**	**7.5%**	$2,007	**$23**	**1.2%**	$917	$844	**92.0%**	$905	$844	93.2%	$3,453	$3,214	**93.1%**	0.424	0.420	$1,029	$967	94.0%	1.26	1.25	2.04	2.00
LogN	10.75	2.7	$6,425	$6,879	**$454**	**7.1%**	$6,494	**$69**	**1.1%**	$2,873	$2,654	**92.4%**	$2,837	$2,654	93.5%	$10,840	$10,120	**93.4%**	0.412	0.409	$3,246	$3,059	94.3%	1.23	1.22	1.93	1.89
LogN	9.63	2.97	$6,803	$7,400	**$596**	**8.8%**	$6,906	**$103**	**1.5%**	$3,444	$3,135	**91.0%**	$3,392	$3,133	92.4%	$12,900	$11,891	**92.2%**	0.458	0.454	$3,788	$3,528	93.1%	1.35	1.34	2.36	2.31
TLogN	10.2	1.95	$173	$190	**$17**	**10.1%**	$176	**$3**	**1.7%**	$91	$79	**86.6%**	$90	$79	88.2%	$345	$311	**90.2%**	0.471	0.449	$97	$86	88.7%	1.84	1.72	5.41	4.64
TLogN	9	2.2	$187	$220	**$33**	**17.7%**	$191	**$4**	**2.1%**	$155	$119	**76.5%**	$152	$119	78.3%	$548	$434	**79.2%**	0.689	0.622	$141	$117	83.4%	3.17	2.66	19.26	13.40
TLogN	10.7	2.385	$1,833	$2,110	**$278**	**15.2%**	$1,881	**$49**	**2.7%**	$1,303	$1,081	**83.0%**	$1,273	$1,080	84.9%	$4,520	$3,900	**86.3%**	0.603	0.574	$1,261	$1,085	86.0%	2.26	2.15	8.95	8.21
TLogN	9.4	2.65	$1,936	$2,417	**$481**	**24.8%**	$1,991	**$56**	**2.9%**	$1,944	$1,402	**72.1%**	$1,883	$1,401	74.4%	$6,771	$5,241	**77.4%**	0.779	0.703	$1,741	$1,381	79.3%	3.02	2.44	17.56	10.37
TLogN	11	2.6	$6,271	$7,382	**$1,111**	**17.7%**	$6,471	**$199**	**3.2%**	$4,923	$3,998	**81.2%**	$4,796	$3,993	83.3%	$18,152	$15,265	**84.1%**	0.650	0.617	$4,638	$3,996	86.2%	2.54	2.37	11.14	9.70
TLogN	10	2.8	$6,333	$7,672	**$1,339**	**21.1%**	$6,343	**$9**	**0.1%**	$5,810	$4,277	**73.6%**	$5,653	$4,277	75.7%	$20,252	$15,400	**76.0%**	0.737	0.674	$5,485	$4,376	79.8%	2.83	2.46	14.86	11.40
	a	**b**																									
Logg	24	2.65	$307	$328	**$22**	**7.0%**	$309	**$2**	**0.7%**	$161	$148	**92.1%**	$159	$148	92.9%	$616	$570	**92.6%**	0.485	0.480	$177	$165	93.0%	1.84	1.81	5.72	5.53
Logg	33	3.3	$308	$322	**$14**	**4.6%**	$307	**-$1**	**-0.4%**	$130	$122	**93.7%**	$129	$122	94.2%	$495	$469	**94.7%**	0.401	0.397	$154	$147	95.0%	1.38	1.36	2.95	2.84
Logg	25	2.5	$1,752	$1,929	**$177**	**10.1%**	$1,799	**$47**	**2.7%**	$1,012	$917	**90.7%**	$996	$916	92.0%	$3,713	$3,395	**91.5%**	0.516	0.509	$1,110	$1,033	93.1%	1.67	1.64	4.71	4.51
Logg	34.5	3.15	$1,491	$1,606	**$115**	**7.7%**	$1,520	**$29**	**2.0%**	$744	$690	**92.7%**	$735	$689	93.7%	$2,764	$2,591	**93.7%**	0.458	0.453	$846	$795	94.1%	1.40	1.38	2.97	2.90
Logg	25.25	2.45	$3,127	$3,489	**$362**	**11.6%**	$3,244	**$118**	**3.8%**	$1,918	$1,735	**90.4%**	$1,883	$1,731	91.9%	$7,221	$6,717	**93.0%**	0.540	0.533	$2,115	$1,951	92.3%	1.66	1.64	4.32	4.21
Logg	34.7	3.07	$2,818	$3,059	**$241**	**8.6%**	$2,888	**$70**	**2.5%**	$1,470	$1,357	**92.3%**	$1,450	$1,355	93.4%	$5,230	$4,902	**93.7%**	0.474	0.469	$1,645	$1,559	94.8%	1.88	1.85	7.63	7.39
TLogg	23.5	2.65	$427	$618	**$192**	**45.0%**	$427	**$1**	**0.2%**	$954	$437	**45.8%**	$935	$437	46.8%	$2,145	$1,241	**57.9%**	1.511	1.023	$451	$299	66.3%	10.07	5.66	160.42	50.99
TLogg	33	3.3	$395	$465	**$70**	**17.7%**	$406	**$12**	**3.0%**	$388	$290	**74.7%**	$382	$289	75.9%	$1,181	$984	**83.4%**	0.821	0.712	$291	$250	85.7%	7.38	5.18	113.19	61.98
TLogg	24.5	2.5	$1,900	$2,550	**$650**	**34.2%**	$1,987	**$87**	**4.6%**	$2,642	$1,628	**61.6%**	$2,561	$1,626	63.5%	$8,394	$5,704	**68.0%**	1.004	0.818	$1,877	$1,462	77.9%	4.21	2.94	28.24	13.90
TLogg	34.5	3.15	$1,683	$1,935	**$252**	**15.0%**	$1,748	**$65**	**3.9%**	$1,244	$1,059	**85.1%**	$1,218	$1,057	86.8%	$4,685	$4,112	**87.8%**	0.629	0.604	$1,324	$1,184	89.4%	1.76	1.65	4.37	3.78
TLogg	24.75	2.45	$3,186	$4,181	**$995**	**31.2%**	$3,333	**$147**	**4.6%**	$4,386	$2,800	**63.8%**	$4,272	$2,797	65.5%	$13,163	$9,326	**70.9%**	1.022	0.839	$3,282	$2,555	77.9%	5.29	3.33	49.11	19.31
TLogg	34.6	3.07	$2,965	$3,469	**$504**	**17.0%**	$3,145	**$180**	**6.1%**	$2,236	$1,903	**85.1%**	$2,178	$1,895	87.0%	$8,111	$7,066	**87.1%**	0.628	0.602	$2,323	$2,062	88.8%	2.50	2.31	13.17	11.29
	ξ	**θ**																									
GPD	0.8	35,000	$667	$944	**$277**	**41.5%**	$697	**$30**	**4.5%**	$1,013	$678	**67.0%**	$974	$678	69.6%	$3,457	$2,408	**69.6%**	1.032	0.972	$822	$594	72.3%	3.44	3.20	17.63	15.28
GPD	0.95	7,500	$726	$1,117	**$392**	**54.0%**	$767	**$42**	**5.8%**	$1,384	$854	**61.7%**	$1,327	$853	64.3%	$4,499	$2,960	**65.8%**	1.188	1.112	$990	$672	67.9%	3.98	3.66	23.72	20.04
GPD	0.875	47,500	$2,031	$2,992	**$961**	**47.3%**	$2,132	**$102**	**5.0%**	$3,459	$2,227	**64.4%**	$3,322	$2,225	67.0%	$11,336	$7,744	**68.3%**	1.110	1.043	$2,654	$1,846	69.5%	3.71	3.44	20.60	17.70
GPD	0.95	25,000	$2,418	$3,720	**$1,302**	**53.8%**	$2,555	**$137**	**5.6%**	$4,610	$2,846	**61.7%**	$4,422	$2,843	64.3%	$14,996	$9,867	**65.8%**	1.189	1.113	$3,292	$2,230	67.7%	3.99	3.67	23.80	20.11
GPD	0.925	50,000	$3,681	$5,550	**$1,869**	**50.8%**	$3,861	**$180**	**4.9%**	$6,704	$4,203	**62.7%**	$6,438	$4,199	65.2%	$21,470	$13,980	**65.1%**	1.160	1.088	$4,936	$3,365	68.2%	3.93	3.63	23.30	19.79
GPD	0.99	27,500	$4,124	$6,457	**$2,332**	**56.5%**	$4,346	**$222**	**5.4%**	$8,272	$4,993	**60.4%**	$7,936	$4,988	62.9%	$26,558	$16,654	**62.7%**	1.229	1.148	$5,761	$3,871	67.2%	4.19	3.84	26.50	22.21
TGPD	0.775	33,500	$603	$812	**$209**	**34.7%**	$606	**$3**	**0.6%**	$790	$538	**68.1%**	$762	$538	70.6%	$3,019	$2,130	**70.6%**	0.939	0.888	$652	$475	72.8%	2.83	2.67	12.10	10.62
TGPD	0.8	25,000	$629	$874	**$245**	**38.8%**	$644	**$14**	**2.3%**	$935	$627	**67.1%**	$903	$627	69.5%	$3,112	$2,203	**70.8%**	1.033	0.974	$715	$525	73.5%	3.45	3.25	18.04	16.12
TGPD	0.8675	50,000	$2,313	$3,366	**$1,052**	**45.5%**	$2,397	**$84**	**3.6%**	$4,111	$2,623	**63.8%**	$3,974	$2,622	66.0%	$11,968	$8,109	**67.8%**	1.181	1.094	$2,823	$1,968	69.7%	5.06	4.53	42.28	34.44
TGPD	0.91	31,000	$2,508	$3,668	**$1,161**	**46.3%**	$2,564	**$56**	**2.2%**	$4,201	$2,632	**62.6%**	$4,037	$2,631	65.2%	$13,671	$9,199	**67.3%**	1.101	1.026	$3,067	$2,124	69.3%	4.57	4.04	42.29	33.55
TGPD	0.92	47,500	$3,953	$5,875	**$1,922**	**48.6%**	$4,088	**$134**	**3.4%**	$7,004	$4,409	**63.0%**	$6,735	$4,407	65.4%	$24,025	$15,841	**65.9%**	1.146	1.078	$5,115	$3,486	68.1%	3.95	3.70	25.63	22.80
TGPD	0.95	35,000	$4,305	$6,619	**$2,313**	**53.7%**	$4,506	**$201**	**4.7%**	$9,696	$5,776	**59.6%**	$9,416	$5,772	61.3%	$25,151	$15,986	**63.6%**	1.423	1.281	$5,692	$3,829	67.3%	8.95	7.54	137.22	101.58

*NOTE: #simulations = 1,000; λ = 50 for 10 years so n ~ 500; α=0.9997

TABLE F7a
RCE vs. LDA for Regulatory Capital Estimation Under iid ($m, λ=75)*

Severity Dist.	Parm1	Parm2	True RCap	Mean MLE RCap	MLE Bias	MLE Bias%	Mean RCE RCap	RCE Bias	RCE Bias%	RMSE MLE RCap	RMSE RCE RCap	RMSE RCap RCE/MLE	StdDev MLE RCap	StdDev RCE RCap	StdDev RCap RCE/MLE	95%CIs MLE RCap	95%CIs RCE RCap	95%CIs RCap RCE/MLE	CV MLE	CV RCE	IQR MLE RCap	IQR RCE RCap	IQR RCap RCE/MLE	Skew MLE RCap	Skew RCE RCap	Kurtosis MLE RCap	Kurtosis RCE RCap
	μ	σ																									
LogN	10	2	$110	$112	$2	1.5%	$110	-$1	-0.5%	$25	$24	97.4%	$25	$24	97.6%	$96	$93	97.5%	0.222	0.221	$33	$32	98.8%	0.75	0.74	1.63	1.61
LogN	7.7	2.55	$103	$106	$3	3.0%	$103	$0	-0.3%	$32	$30	95.9%	$32	$30	96.4%	$120	$115	95.8%	0.297	0.296	$40	$39	97.0%	1.04	1.03	2.88	2.83
LogN	10.4	2.5	$1,250	$1,285	$35	2.8%	$1,246	-$4	-0.3%	$375	$360	96.1%	$373	$360	96.5%	$1,422	$1,365	95.9%	0.291	0.289	$477	$464	97.3%	1.01	1.00	2.75	2.70
LogN	9.27	2.77	$1,236	$1,281	$45	3.7%	$1,234	-$2	-0.1%	$420	$400	95.2%	$418	$400	95.8%	$1,582	$1,508	95.4%	0.326	0.324	$528	$508	96.2%	1.16	1.15	3.52	3.45
LogN	10.75	2.7	$4,059	$4,198	$139	3.4%	$4,051	-$7	-0.2%	$1,338	$1,277	95.5%	$1,330	$1,277	96.0%	$5,045	$4,819	95.5%	0.317	0.315	$1,685	$1,626	96.5%	1.12	1.11	3.31	3.24
LogN	9.63	2.97	$4,074	$4,252	$177	4.4%	$4,075	$1	0.0%	$1,509	$1,427	94.6%	$1,498	$1,427	95.3%	$5,646	$5,355	94.8%	0.352	0.350	$1,874	$1,791	95.6%	1.28	1.26	4.19	4.10
TLogN	10.2	1.95	$133	$138	$5	3.6%	$132	-$1	-0.8%	$45	$42	92.9%	$45	$42	93.4%	$174	$160	92.4%	0.323	0.315	$55	$51	93.6%	1.14	1.09	2.22	2.04
TLogN	9	2.2	$136	$146	$10	7.3%	$136	-$1	-0.5%	$65	$56	87.1%	$64	$56	88.2%	$241	$212	88.1%	0.437	0.416	$73	$66	89.6%	1.56	1.45	3.63	3.00
TLogN	10.7	2.385	$1,251	$1,327	$76	6.0%	$1,244	-$7	-0.6%	$543	$491	90.5%	$538	$491	91.3%	$2,168	$1,988	91.7%	0.405	0.395	$616	$569	92.3%	1.28	1.24	2.18	2.03
TLogN	9.4	2.65	$1,259	$1,434	$175	13.9%	$1,290	$31	2.4%	$798	$666	83.4%	$779	$665	85.4%	$2,716	$2,337	86.1%	0.543	0.515	$862	$749	86.9%	2.03	1.89	7.26	6.35
TLogN	11	2.6	$4,079	$4,456	$377	9.2%	$4,130	$51	1.3%	$2,049	$1,814	88.6%	$2,014	$1,814	90.1%	$7,870	$7,053	89.6%	0.452	0.439	$2,323	$2,120	91.3%	1.62	1.57	4.15	3.88
TLogN	10	2.8	$3,978	$4,505	$527	13.2%	$4,046	$68	1.7%	$2,606	$2,194	84.2%	$2,552	$2,193	85.9%	$9,400	$8,172	86.9%	0.567	0.542	$2,836	$2,487	87.7%	2.05	1.94	7.51	6.72
	a	b																									
Logg	24	2.65	$184	$192	$8	4.4%	$185	$1	0.8%	$68	$64	95.2%	$67	$64	95.9%	$256	$246	96.0%	0.350	0.348	$84	$80	95.5%	1.06	1.06	1.89	1.88
Logg	33	3.3	$198	$205	$6	3.2%	$199	$1	0.3%	$64	$61	96.1%	$63	$61	96.6%	$236	$228	96.4%	0.310	0.308	$82	$80	96.9%	0.99	0.98	1.69	1.64
Logg	25	2.5	$1,004	$1,062	$58	5.8%	$1,019	$16	1.5%	$427	$404	94.6%	$424	$404	95.4%	$1,616	$1,543	95.5%	0.399	0.396	$516	$497	96.3%	1.16	1.15	2.00	1.98
Logg	34.5	3.15	$925	$962	$37	4.0%	$932	$7	0.7%	$316	$302	95.6%	$313	$302	96.3%	$1,232	$1,183	96.0%	0.326	0.324	$401	$390	97.2%	0.93	0.93	1.63	1.60
Logg	25.25	2.45	$1,765	$1,857	$92	5.2%	$1,780	$15	0.8%	$760	$718	94.4%	$754	$718	95.1%	$2,946	$2,796	94.9%	0.406	0.403	$948	$902	95.1%	1.15	1.14	1.95	1.91
Logg	34.7	3.07	$1,720	$1,777	$58	3.3%	$1,719	-$1	-0.1%	$615	$588	95.6%	$612	$588	96.1%	$2,424	$2,340	96.5%	0.344	0.342	$766	$739	96.4%	1.07	1.06	1.59	1.58
TLogg	23.5	2.65	$261	$299	$38	14.6%	$258	-$3	-1.0%	$211	$162	76.6%	$208	$162	77.9%	$723	$587	81.2%	0.694	0.625	$196	$165	84.1%	2.73	2.39	11.50	9.10
TLogg	33	3.3	$257	$277	$20	7.7%	$257	$0	0.1%	$134	$119	88.3%	$133	$119	89.3%	$489	$439	89.8%	0.481	0.462	$146	$132	90.7%	1.87	1.72	7.04	5.75
TLogg	24.5	2.5	$1,104	$1,290	$186	16.9%	$1,138	$34	3.1%	$920	$740	80.4%	$901	$739	82.0%	$3,334	$2,762	82.9%	0.699	0.649	$860	$737	85.6%	2.38	2.09	8.63	6.25
TLogg	34.5	3.15	$1,049	$1,130	$81	7.7%	$1,062	$13	1.2%	$506	$459	90.6%	$500	$459	91.8%	$1,873	$1,740	92.9%	0.443	0.432	$593	$552	93.0%	1.34	1.30	2.80	2.60
TLogg	24.75	2.45	$1,820	$2,111	$291	16.0%	$1,879	$59	3.2%	$1,390	$1,153	83.0%	$1,359	$1,151	84.7%	$5,320	$4,437	83.4%	0.644	0.613	$1,365	$1,196	87.7%	1.98	1.88	5.86	5.30
TLogg	34.6	3.07	$1,817	$1,967	$150	8.3%	$1,852	$36	2.0%	$893	$812	91.0%	$880	$812	92.2%	$3,313	$3,053	92.2%	0.448	0.438	$1,034	$952	92.1%	1.51	1.45	4.01	3.65
	ξ	θ																									
GPD	0.8	35,000	$361	$424	$63	17.6%	$360	$0	-0.1%	$278	$223	80.4%	$271	$223	82.6%	$1,035	$843	81.4%	0.638	0.620	$313	$262	83.8%	1.75	1.71	4.25	4.05
GPD	0.95	7,500	$344	$423	$79	23.0%	$345	$1	0.4%	$318	$244	76.8%	$308	$244	79.3%	$1,170	$923	78.9%	0.728	0.706	$332	$265	79.8%	2.01	1.96	5.93	5.65
GPD	0.875	47,500	$1,027	$1,232	$206	20.0%	$1,027	$0	0.0%	$867	$681	78.6%	$842	$681	80.9%	$3,180	$2,555	80.3%	0.683	0.663	$932	$764	82.0%	1.88	1.83	5.07	4.82
GPD	0.95	25,000	$1,146	$1,407	$261	22.8%	$1,149	$3	0.2%	$1,057	$812	76.8%	$1,024	$812	79.3%	$3,900	$3,077	78.9%	0.728	0.706	$1,103	$881	79.9%	2.02	1.97	5.99	5.71
GPD	0.925	50,000	$1,782	$2,170	$388	21.8%	$1,784	$2	0.1%	$1,595	$1,235	77.4%	$1,547	$1,235	79.8%	$5,867	$4,647	79.2%	0.713	0.692	$1,679	$1,344	80.1%	1.97	1.93	5.69	5.41
GPD	0.99	27,500	$1,889	$2,347	$458	24.3%	$1,896	$7	0.4%	$1,822	$1,384	75.9%	$1,763	$1,384	78.5%	$6,780	$5,254	77.5%	0.751	0.730	$1,862	$1,467	78.8%	2.10	2.05	6.56	6.25
TGPD	0.775	33,500	$334	$393	$59	17.7%	$333	$0	-0.1%	$276	$220	80.0%	$269	$220	81.9%	$994	$814	81.9%	0.685	0.661	$270	$226	83.9%	2.50	2.42	10.67	9.98
TGPD	0.8	25,000	$341	$408	$67	19.6%	$343	$2	0.7%	$301	$238	79.0%	$293	$238	81.0%	$1,039	$844	81.3%	0.720	0.693	$280	$232	83.1%	2.89	2.71	17.85	15.44
TGPD	0.8675	50,000	$1,178	$1,421	$243	20.6%	$1,174	-$4	-0.3%	$1,129	$877	77.7%	$1,102	$877	79.6%	$3,765	$3,012	80.0%	0.776	0.747	$994	$806	81.1%	3.69	3.56	27.76	26.34
TGPD	0.91	31,000	$1,231	$1,516	$285	23.2%	$1,236	$6	0.5%	$1,193	$913	76.5%	$1,159	$912	78.8%	$4,039	$3,183	78.8%	0.764	0.738	$1,157	$927	80.1%	2.45	2.36	10.07	9.29
TGPD	0.92	47,500	$1,923	$2,383	$461	24.0%	$1,940	$17	0.9%	$1,969	$1,504	76.4%	$1,915	$1,504	78.6%	$6,545	$5,202	79.5%	0.803	0.776	$1,733	$1,377	79.5%	3.12	3.03	19.22	18.38
TGPD	0.95	35,000	$2,041	$2,542	$501	24.6%	$2,050	$9	0.4%	$2,066	$1,562	75.6%	$2,004	$1,562	77.9%	$7,470	$5,817	77.9%	0.789	0.762	$1,911	$1,507	78.8%	2.34	2.26	8.34	7.85

*NOTE: #simulations = 1,000; λ = 75 for 10 years so n ~ 750; α=0.999

TABLE F7b

RCE vs. LDA for Economic Capital Estimation Under iid ($m, λ=75)*

Severity Dist.	Parm1	Parm2	True ECap	Mean MLE ECap	MLE Bias	MLE Bias%	Mean RCE ECap	RCE Bias	RCE Bias%	RMSE MLE ECap	RMSE RCE ECap	RMSE ECap RCE/MLE	StdDev MLE ECap	StdDev RCE ECap	StdDev ECap RCE/MLE	95%CIs MLE ECap	95%CIs RCE ECap	95%CIs ECap RCE/MLE	CV MLE	CV RCE	IQR MLE ECap	IQR RCE ECap	IQR ECap RCE/MLE	Skew MLE ECap	Skew RCE ECap	Kurtosis MLE ECap	Kurtosis RCE ECap
	μ	σ																									
LogN	10	2	$179	$182	$3	1.8%	$178	-$1	-0.4%	$44	$42	97.2%	$44	$42	97.5%	$166	$162	97.4%	0.240	0.238	$57	$56	97.6%	0.82	0.81	1.93	1.90
LogN	7.7	2.55	$199	$206	$7	3.4%	$199	$0	-0.1%	$66	$63	95.6%	$65	$63	96.1%	$246	$236	96.0%	0.317	0.315	$83	$80	96.3%	1.13	1.12	3.41	3.35
LogN	10.4	2.5	$2,372	$2,449	$77	3.2%	$2,370	-$2	-0.1%	$763	$731	95.8%	$759	$731	96.2%	$2,869	$2,757	96.1%	0.310	0.308	$968	$934	96.4%	1.10	1.09	3.25	3.19
LogN	9.27	2.77	$2,534	$2,641	$107	4.2%	$2,537	$3	0.1%	$923	$876	94.9%	$917	$876	95.5%	$3,444	$3,284	95.4%	0.347	0.345	$1,153	$1,102	95.6%	1.27	1.25	4.18	4.09
LogN	10.75	2.7	$8,160	$8,482	$322	3.9%	$8,165	$5	0.1%	$2,881	$2,740	95.1%	$2,863	$2,740	95.7%	$10,772	$10,292	95.6%	0.338	0.336	$3,613	$3,462	95.8%	1.22	1.21	3.92	3.84
LogN	9.63	2.97	$8,838	$9,278	$441	5.0%	$8,864	$26	0.3%	$3,508	$3,303	94.2%	$3,480	$3,303	94.9%	$13,001	$12,320	94.8%	0.375	0.373	$4,320	$4,105	95.0%	1.40	1.38	4.99	4.88
TLogN	10.2	1.95	$209	$218	$10	4.6%	$207	-$1	-0.6%	$80	$73	91.8%	$79	$73	92.5%	$306	$284	92.7%	0.364	0.354	$95	$89	93.4%	1.28	1.22	2.78	2.55
TLogN	9	2.2	$228	$250	$22	9.5%	$228	$0	0.0%	$127	$108	84.9%	$125	$108	86.2%	$460	$403	87.5%	0.498	0.471	$138	$121	87.7%	1.75	1.61	4.77	3.81
TLogN	10.7	2.385	$2,266	$2,433	$167	7.4%	$2,259	-$7	-0.3%	$1,098	$979	89.2%	$1,085	$979	90.2%	$4,365	$3,960	90.7%	0.446	0.433	$1,233	$1,121	90.9%	1.39	1.35	2.58	2.40
TLogN	9.4	2.65	$2,435	$2,849	$414	17.0%	$2,513	$77	3.2%	$1,781	$1,440	80.9%	$1,733	$1,438	83.0%	$5,938	$4,994	84.1%	0.608	0.572	$1,827	$1,577	86.3%	2.30	2.11	9.09	7.76
TLogN	11	2.6	$7,882	$8,751	$869	11.0%	$8,024	$142	1.8%	$4,434	$3,865	87.2%	$4,348	$3,862	88.8%	$17,033	$15,067	88.5%	0.497	0.481	$4,824	$4,397	91.1%	1.77	1.70	4.92	4.55
TLogN	10	2.8	$8,072	$9,372	$1,300	16.1%	$8,266	$194	2.4%	$6,039	$4,945	81.9%	$5,897	$4,941	83.8%	$21,344	$18,081	84.7%	0.629	0.598	$6,260	$5,420	86.6%	2.28	2.14	9.32	8.18
	a	b																									
Logg	24	2.65	$401	$423	$22	5.4%	$406	$5	1.2%	$164	$155	94.6%	$163	$155	95.4%	$618	$590	95.5%	0.384	0.381	$201	$192	95.5%	1.17	1.16	2.26	2.24
Logg	33	3.3	$392	$407	$16	4.0%	$394	$3	0.6%	$139	$133	95.6%	$139	$133	96.2%	$512	$497	97.0%	0.340	0.338	$178	$171	96.0%	1.09	1.08	2.05	1.99
Logg	25	2.5	$2,337	$2,501	$164	7.0%	$2,389	$52	2.2%	$1,105	$1,037	93.9%	$1,092	$1,036	94.8%	$4,134	$3,945	95.4%	0.437	0.434	$1,299	$1,241	95.5%	1.26	1.25	2.40	2.36
Logg	34.5	3.15	$1,922	$2,015	$93	4.8%	$1,943	$22	1.1%	$723	$688	95.1%	$717	$688	95.9%	$2,800	$2,687	95.9%	0.356	0.354	$919	$879	95.6%	1.03	1.02	1.99	1.96
Logg	25.25	2.45	$4,198	$4,467	$268	6.4%	$4,258	$60	1.4%	$2,003	$1,878	93.7%	$1,985	$1,877	94.5%	$7,800	$7,332	94.0%	0.444	0.441	$2,419	$2,293	94.8%	1.25	1.24	2.34	2.29
Logg	34.7	3.07	$3,657	$3,808	$152	4.1%	$3,667	$10	0.3%	$1,443	$1,372	95.1%	$1,435	$1,372	95.6%	$5,710	$5,468	95.8%	0.377	0.374	$1,782	$1,704	95.7%	1.15	1.14	1.85	1.83
TLogg	23.5	2.65	$555	$666	$111	20.1%	$547	-$8	-1.4%	$558	$389	69.8%	$546	$389	71.3%	$1,827	$1,392	76.2%	0.820	0.711	$476	$377	79.3%	3.16	2.64	14.87	10.80
TLogg	33	3.3	$501	$553	$52	10.3%	$505	$4	0.8%	$310	$266	85.9%	$306	$266	87.1%	$1,111	$981	88.2%	0.553	0.527	$318	$285	89.7%	2.21	1.99	10.14	7.90
TLogg	24.5	2.5	$2,523	$3,089	$566	22.4%	$2,632	$109	4.3%	$2,570	$1,944	75.6%	$2,507	$1,941	77.4%	$9,207	$7,231	78.5%	0.812	0.738	$2,184	$1,852	84.8%	2.81	2.34	12.31	7.90
TLogg	34.5	3.15	$2,167	$2,380	$213	9.8%	$2,210	$42	2.0%	$1,202	$1,070	89.0%	$1,183	$1,069	90.4%	$4,360	$4,012	92.0%	0.497	0.484	$1,365	$1,263	92.5%	1.52	1.46	3.67	3.38
TLogg	24.75	2.45	$4,259	$5,138	$879	20.6%	$4,444	$185	4.3%	$3,867	$3,073	79.5%	$3,766	$3,067	81.4%	$14,868	$11,839	79.6%	0.733	0.690	$3,562	$3,002	84.3%	2.23	2.09	7.40	6.56
TLogg	34.6	3.07	$3,843	$4,239	$396	10.3%	$3,948	$104	2.7%	$2,161	$1,933	89.4%	$2,125	$1,930	90.8%	$7,924	$7,205	90.9%	0.501	0.489	$2,456	$2,218	90.3%	1.71	1.64	5.25	4.73
	ξ	θ																									
GPD	0.8	35,000	$924	$1,139	$215	23.3%	$927	$3	0.4%	$868	$664	76.4%	$841	$664	78.9%	$3,207	$2,494	77.8%	0.738	0.716	$920	$734	79.9%	1.99	1.94	5.58	5.28
GPD	0.95	7,500	$1,067	$1,389	$322	30.2%	$1,076	$10	0.9%	$1,207	$872	72.2%	$1,163	$872	75.0%	$4,372	$3,251	74.4%	0.837	0.810	$1,165	$896	76.9%	2.30	2.23	7.80	7.34
GPD	0.875	47,500	$2,897	$3,665	$767	26.5%	$2,912	$14	0.5%	$2,987	$2,221	74.3%	$2,887	$2,221	76.9%	$10,866	$8,327	76.6%	0.788	0.763	$3,005	$2,362	78.6%	2.15	2.09	6.66	6.27
GPD	0.95	25,000	$3,556	$4,622	$1,066	30.0%	$3,582	$26	0.7%	$4,015	$2,902	72.3%	$3,870	$2,902	75.0%	$14,574	$10,836	74.4%	0.837	0.810	$3,870	$2,985	77.2%	2.31	2.24	7.87	7.40
GPD	0.925	50,000	$5,358	$6,898	$1,539	28.7%	$5,391	$33	0.6%	$5,867	$4,281	73.0%	$5,661	$4,281	75.6%	$21,129	$15,859	75.1%	0.821	0.794	$5,732	$4,409	76.9%	2.26	2.19	7.47	7.03
GPD	0.99	27,500	$6,163	$8,127	$1,964	31.9%	$6,214	$51	0.8%	$7,291	$5,192	71.2%	$7,021	$5,192	73.9%	$26,759	$19,663	73.5%	0.864	0.836	$6,967	$5,190	74.5%	2.40	2.33	8.64	8.10
TGPD	0.775	33,500	$826	$1,024	$198	23.9%	$830	$4	0.4%	$851	$645	75.8%	$828	$645	78.0%	$2,988	$2,326	77.8%	0.808	0.777	$780	$613	78.7%	2.95	2.83	14.66	13.57
TGPD	0.8	25,000	$872	$1,103	$231	26.5%	$885	$13	1.5%	$968	$722	74.6%	$940	$722	76.8%	$3,242	$2,506	77.3%	0.852	0.816	$825	$655	79.4%	3.62	3.34	27.96	23.58
TGPD	0.8675	50,000	$3,291	$4,203	$912	27.7%	$3,302	$11	0.3%	$3,988	$2,921	73.3%	$3,882	$2,921	75.3%	$12,841	$9,740	75.9%	0.924	0.885	$3,175	$2,433	76.6%	4.54	4.37	40.78	38.39
TGPD	0.91	31,000	$3,628	$4,751	$1,122	30.9%	$3,672	$44	1.2%	$4,379	$3,144	71.8%	$4,233	$3,144	74.3%	$14,211	$10,598	74.6%	0.891	0.856	$3,889	$2,951	75.9%	2.91	2.77	14.08	12.78
TGPD	0.92	47,500	$5,743	$7,581	$1,837	32.0%	$5,844	$101	1.8%	$7,385	$5,298	71.7%	$7,153	$5,297	74.1%	$23,816	$17,844	74.9%	0.944	0.906	$5,940	$4,474	75.3%	3.76	3.62	27.63	26.07
TGPD	0.95	35,000	$6,330	$8,404	$2,073	32.8%	$6,406	$75	1.2%	$7,964	$5,636	70.8%	$7,689	$5,635	73.3%	$28,569	$20,769	72.7%	0.915	0.880	$6,704	$5,060	75.5%	2.71	2.61	11.34	10.46

*NOTE: #simulations = 1,000; λ = 75 for 10 years so n ~ 750; α=0.9997

TABLE F8a

RCE vs. LDA for Regulatory Capital Estimation Under iid ($m, λ=100)*

Severity Dist.	Parm1	Parm2	True RCap	Mean MLE RCap	MLE Bias	MLE Bias%	Mean RCE RCap	RCE Bias	RCE Bias%	RMSE MLE RCap	RMSE RCE RCap	RMSE RCap RCE/MLE	StdDev MLE RCap	StdDev RCE RCap	StdDev RCap RCE/MLE	95%CIs MLE RCap	95%CIs RCE RCap	95%CIs RCap RCE/MLE	CV MLE	CV RCE	IQR MLE RCap	IQR RCE RCap	IQR RCap RCE/MLE	Skew MLE RCap	Skew RCE RCap	Kurtosis MLE RCap	Kurtosis RCE RCap
	μ	σ																									
LogN	10	2	$128	$130	$2	1.5%	$128	$0	-0.1%	$25	$25	97.7%	$25	$25	97.9%	$100	$98	98.2%	0.195	0.194	$32	$32	98.4%	0.75	0.74	1.07	1.07
LogN	7.7	2.55	$122	$126	$3	2.8%	$123	$0	0.1%	$33	$32	96.4%	$33	$32	96.9%	$130	$126	96.7%	0.263	0.262	$41	$40	97.6%	0.96	0.95	1.69	1.69
LogN	10.4	2.5	$1,478	$1,518	$40	2.7%	$1,479	$1	0.1%	$392	$378	96.5%	$390	$378	97.0%	$1,534	$1,485	96.8%	0.257	0.256	$484	$473	97.7%	0.94	0.93	1.63	1.62
LogN	9.27	2.77	$1,483	$1,534	$51	3.4%	$1,487	$3	0.2%	$446	$427	95.8%	$443	$427	96.4%	$1,740	$1,674	96.2%	0.289	0.287	$543	$526	97.0%	1.04	1.04	1.99	1.98
LogN	10.75	2.7	$4,852	$5,010	$157	3.2%	$4,861	$9	0.2%	$1,414	$1,357	96.0%	$1,405	$1,357	96.6%	$5,522	$5,322	96.4%	0.280	0.279	$1,727	$1,679	97.2%	1.01	1.01	1.89	1.88
LogN	9.63	2.97	$4,949	$5,148	$200	4.0%	$4,966	$18	0.4%	$1,619	$1,542	95.2%	$1,606	$1,542	96.0%	$6,303	$6,037	95.8%	0.312	0.310	$1,953	$1,883	96.4%	1.12	1.12	2.28	2.27
TLogN	10.2	1.95	$155	$159	$4	2.9%	$154	-$1	-0.4%	$43	$41	94.5%	$43	$41	95.0%	$165	$158	95.3%	0.270	0.265	$53	$50	95.7%	1.06	1.03	1.68	1.58
TLogN	9	2.2	$159	$168	$8	5.3%	$158	-$1	-0.4%	$63	$57	90.4%	$63	$57	91.2%	$250	$226	90.5%	0.374	0.361	$71	$66	92.1%	1.27	1.23	1.82	1.67
TLogN	10.7	2.385	$1,470	$1,547	$77	5.2%	$1,472	$2	0.2%	$573	$529	92.4%	$568	$529	93.3%	$2,204	$2,053	93.1%	0.367	0.360	$683	$645	94.3%	1.28	1.25	2.19	2.10
TLogN	9.4	2.65	$1,496	$1,621	$125	8.4%	$1,498	$2	0.1%	$742	$652	87.8%	$731	$652	89.1%	$2,809	$2,538	90.3%	0.451	0.435	$843	$763	90.5%	1.62	1.54	4.21	3.78
TLogN	11	2.6	$4,842	$5,114	$272	5.6%	$4,829	-$12	-0.3%	$1,963	$1,801	91.8%	$1,944	$1,801	92.7%	$7,232	$6,704	92.7%	0.380	0.373	$2,431	$2,250	92.6%	1.07	1.05	1.27	1.21
TLogN	10	2.8	$4,767	$5,184	$418	8.8%	$4,781	$14	0.3%	$2,601	$2,291	88.1%	$2,567	$2,291	89.2%	$9,703	$8,673	89.4%	0.495	0.479	$2,923	$2,641	90.4%	1.94	1.84	7.94	7.01
	a	b																									
Logg	24	2.65	$224	$226	$3	1.1%	$224	$0	0.0%	$68	$67	98.6%	$68	$67	98.6%	$269	$265	98.6%	0.302	0.302	$90	$88	98.2%	0.80	0.80	0.62	0.62
Logg	33	3.3	$237	$238	$1	0.4%	$236	-$1	-0.6%	$62	$61	98.9%	$62	$61	98.9%	$241	$238	98.8%	0.259	0.259	$81	$80	98.9%	0.70	0.70	0.51	0.50
Logg	25	2.5	$1,238	$1,254	$16	1.3%	$1,238	$0	0.0%	$401	$395	98.5%	$401	$395	98.5%	$1,539	$1,521	98.8%	0.319	0.319	$518	$509	98.2%	0.93	0.93	1.01	1.01
Logg	34.5	3.15	$1,116	$1,122	$7	0.6%	$1,110	-$5	-0.5%	$314	$310	98.8%	$314	$310	98.8%	$1,189	$1,175	98.8%	0.280	0.279	$401	$397	98.9%	0.83	0.83	1.10	1.10
Logg	25.25	2.45	$2,188	$2,223	$35	1.6%	$2,193	$5	0.2%	$749	$737	98.4%	$748	$737	98.5%	$2,830	$2,794	98.7%	0.336	0.336	$945	$930	98.5%	1.01	1.01	1.55	1.55
Logg	34.7	3.07	$2,083	$2,092	$9	0.4%	$2,069	-$14	-0.7%	$596	$589	98.7%	$596	$588	98.7%	$2,225	$2,197	98.7%	0.285	0.284	$748	$736	98.4%	0.90	0.90	1.26	1.25
TLogg	23.5	2.65	$317	$360	$43	13.5%	$324	$7	2.1%	$244	$197	80.6%	$240	$197	81.9%	$798	$672	84.2%	0.668	0.608	$219	$191	87.2%	4.40	3.52	41.30	26.74
TLogg	33	3.3	$307	$327	$20	6.6%	$310	$3	0.9%	$147	$134	91.2%	$145	$134	92.0%	$577	$531	92.0%	0.444	0.431	$162	$150	92.9%	1.63	1.58	3.99	3.71
TLogg	24.5	2.5	$1,357	$1,492	$135	9.9%	$1,366	$9	0.6%	$872	$757	86.9%	$862	$757	87.9%	$3,102	$2,732	88.1%	0.577	0.555	$907	$817	90.0%	2.03	1.92	6.67	5.93
TLogg	34.5	3.15	$1,264	$1,325	$61	4.8%	$1,265	$1	0.0%	$529	$493	93.3%	$525	$493	93.9%	$2,018	$1,900	94.2%	0.397	0.390	$637	$601	94.4%	1.39	1.37	3.38	3.27
TLogg	24.75	2.45	$2,249	$2,471	$222	9.9%	$2,270	$21	0.9%	$1,446	$1,266	87.5%	$1,429	$1,266	88.6%	$5,816	$5,118	88.0%	0.578	0.558	$1,526	$1,369	89.7%	1.96	1.88	5.89	5.35
TLogg	34.6	3.07	$2,198	$2,294	$95	4.3%	$2,192	-$6	-0.3%	$907	$849	93.6%	$902	$849	94.1%	$3,487	$3,273	93.9%	0.393	0.387	$1,099	$1,038	94.4%	1.25	1.23	2.11	2.05
	ξ	θ																									
GPD	0.8	35,000	$455	$519	$64	14.0%	$457	$2	0.3%	$315	$266	84.2%	$309	$266	86.0%	$1,132	$982	86.8%	0.596	0.582	$336	$292	86.9%	2.01	1.97	6.63	6.35
GPD	0.95	7,500	$452	$534	$82	18.2%	$456	$4	0.8%	$374	$304	81.2%	$365	$304	83.2%	$1,264	$1,064	84.1%	0.683	0.667	$378	$316	83.7%	2.33	2.28	9.05	8.69
GPD	0.875	47,500	$1,322	$1,536	$213	16.1%	$1,331	$9	0.7%	$1,006	$832	82.7%	$983	$832	84.6%	$3,491	$2,992	85.7%	0.640	0.625	$1,045	$895	85.6%	2.15	2.10	7.68	7.34
GPD	0.95	25,000	$1,507	$1,784	$277	18.3%	$1,521	$14	0.9%	$1,250	$1,015	81.2%	$1,219	$1,014	83.2%	$4,214	$3,546	84.1%	0.683	0.667	$1,261	$1,058	83.9%	2.32	2.27	8.98	8.62
GPD	0.925	50,000	$2,327	$2,730	$403	17.3%	$2,341	$14	0.6%	$1,866	$1,525	81.7%	$1,822	$1,525	83.7%	$6,360	$5,351	84.1%	0.667	0.651	$1,900	$1,606	84.5%	2.28	2.22	8.68	8.31
GPD	0.99	27,500	$2,512	$2,997	$484	19.3%	$2,534	$22	0.9%	$2,168	$1,745	80.5%	$2,113	$1,745	82.6%	$7,218	$6,034	83.6%	0.705	0.689	$2,153	$1,786	83.0%	2.43	2.38	9.94	9.59
TGPD	0.775	33,500	$418	$479	$60	14.4%	$421	$2	0.6%	$289	$242	83.7%	$283	$242	85.6%	$1,019	$871	85.5%	0.591	0.576	$312	$276	88.3%	2.15	2.09	8.19	7.75
TGPD	0.8	25,000	$430	$499	$69	16.0%	$436	$6	1.4%	$297	$246	83.0%	$289	$246	85.3%	$1,123	$961	85.6%	0.579	0.565	$325	$278	85.7%	1.71	1.66	4.97	4.61
TGPD	0.8675	50,000	$1,514	$1,753	$239	15.8%	$1,511	-$3	-0.2%	$1,098	$901	82.1%	$1,071	$901	84.1%	$4,063	$3,418	84.1%	0.611	0.596	$1,189	$1,007	84.7%	1.76	1.71	4.52	4.28
TGPD	0.91	31,000	$1,600	$1,885	$285	17.8%	$1,609	$9	0.6%	$1,207	$978	81.0%	$1,173	$978	83.4%	$4,254	$3,544	83.3%	0.622	0.608	$1,361	$1,138	83.6%	1.65	1.61	4.76	4.46
TGPD	0.92	47,500	$2,508	$2,937	$429	17.1%	$2,501	-$6	-0.2%	$1,982	$1,604	80.9%	$1,935	$1,604	82.9%	$6,927	$5,740	82.9%	0.659	0.641	$2,025	$1,683	83.1%	2.22	2.16	8.73	8.24
TGPD	0.95	35,000	$2,684	$3,196	$512	19.1%	$2,703	$19	0.7%	$2,240	$1,796	80.2%	$2,181	$1,796	82.4%	$8,066	$6,654	82.5%	0.682	0.665	$2,389	$1,991	83.3%	1.95	1.89	5.31	5.03

*NOTE: #simulations = 1,000; λ = 100 for 10 years so n ~ 1,000; α=0.999

TABLE F8b

RCE vs. LDA for Economic Capital Estimation Under iid ($m, λ=100)*

Severity Dist.	Parm1	Parm2	True ECap	Mean MLE ECap	MLE Bias	MLE Bias%	Mean RCE ECap	RCE Bias	RCE Bias%	RMSE MLE ECap	RMSE RCE ECap	RMSE ECap RCE/MLE	StdDev MLE ECap	StdDev RCE ECap	StdDev ECap RCE/MLE	95%CIs MLE ECap	95%CIs RCE ECap	95%CIs ECap RCE/MLE	CV MLE	CV RCE	IQR MLE ECap	IQR RCE ECap	IQR ECap RCE/MLE	Skew MLE ECap	Skew RCE ECap	Kurtosis MLE ECap	Kurtosis RCE ECap
	μ	**σ**																									
LogN	10	2	$204	$208	**$4**	**1.8%**	$204	**$0**	**0.0%**	$44	$43	**97.5%**	$44	$43	97.8%	$173	$169	**97.6%**	0.211	0.210	$56	$55	98.0%	0.79	0.79	1.19	1.19
LogN	7.7	2.55	$233	$241	**$8**	**3.3%**	$234	**$1**	**0.3%**	$68	$65	**96.1%**	$67	$65	96.7%	$266	$257	**96.5%**	0.280	0.279	$83	$81	97.4%	1.01	1.01	1.88	1.87
LogN	10.4	2.5	$2,773	$2,860	**$86**	**3.1%**	$2,781	**$7**	**0.3%**	$789	$759	**96.2%**	$784	$759	96.8%	$3,098	$2,992	**96.6%**	0.274	0.273	$967	$943	97.5%	0.99	0.99	1.81	1.80
LogN	9.27	2.77	$3,008	$3,126	**$118**	**3.9%**	$3,021	**$13**	**0.4%**	$968	$924	**95.4%**	$961	$924	96.2%	$3,787	$3,635	**96.0%**	0.307	0.306	$1,171	$1,133	96.7%	1.10	1.10	2.21	2.20
LogN	10.75	2.7	$9,647	$10,005	**$358**	**3.7%**	$9,685	**$38**	**0.4%**	$3,011	$2,880	**95.6%**	$2,990	$2,880	96.3%	$11,789	$11,334	**96.1%**	0.299	0.297	$3,655	$3,543	96.9%	1.07	1.07	2.10	2.09
LogN	9.63	2.97	$10,612	$11,100	**$488**	**4.6%**	$10,675	**$63**	**0.6%**	$3,716	$3,524	**94.8%**	$3,683	$3,523	95.7%	$14,489	$13,836	**95.5%**	0.332	0.330	$4,442	$4,273	96.2%	1.19	1.18	2.54	2.52
TLogN	10.2	1.95	$239	$247	**$9**	**3.7%**	$238	**-$1**	**-0.2%**	$76	$71	**93.7%**	$75	$71	94.3%	$287	$272	**94.5%**	0.305	0.299	$92	$87	94.5%	1.17	1.13	2.10	1.97
TLogN	9	2.2	$263	$281	**$18**	**6.9%**	$263	**$0**	**-0.1%**	$121	$107	**88.8%**	$120	$107	89.9%	$481	$429	**89.3%**	0.425	0.409	$136	$122	90.0%	1.38	1.33	2.21	2.01
TLogN	10.7	2.385	$2,631	$2,799	**$168**	**6.4%**	$2,645	**$14**	**0.5%**	$1,143	$1,045	**91.5%**	$1,131	$1,045	92.5%	$4,407	$4,064	**92.2%**	0.404	0.395	$1,339	$1,249	93.3%	1.38	1.34	2.56	2.44
TLogN	9.4	2.65	$2,861	$3,156	**$295**	**10.3%**	$2,876	**$15**	**0.5%**	$1,611	$1,385	**86.0%**	$1,584	$1,385	87.4%	$6,059	$5,313	**87.7%**	0.502	0.482	$1,767	$1,583	89.6%	1.79	1.70	5.20	4.61
TLogN	11	2.6	$9,252	$9,877	**$625**	**6.8%**	$9,254	**$1**	**0.0%**	$4,136	$3,752	**90.7%**	$4,088	$3,752	91.8%	$15,245	$13,984	**91.7%**	0.414	0.406	$5,101	$4,697	92.1%	1.15	1.12	1.52	1.44
TLogN	10	2.8	$9,568	$10,598	**$1,031**	**10.8%**	$9,644	**$77**	**0.8%**	$5,901	$5,092	**86.3%**	$5,810	$5,091	87.6%	$21,708	$19,140	**88.2%**	0.548	0.528	$6,394	$5,671	88.7%	2.20	2.06	10.32	8.94
	a	**b**																									
Logg	24	2.65	$485	$494	**$8**	**1.7%**	$487	**$2**	**0.4%**	$164	$161	**98.4%**	$163	$161	98.5%	$643	$632	**98.4%**	0.331	0.330	$210	$207	98.5%	0.87	0.87	0.77	0.77
Logg	33	3.3	$464	$468	**$3**	**0.7%**	$463	**-$1**	**-0.3%**	$133	$131	**98.8%**	$133	$131	98.8%	$520	$513	**98.7%**	0.284	0.284	$172	$170	98.8%	0.77	0.76	0.64	0.63
Logg	25	2.5	$2,862	$2,915	**$53**	**1.9%**	$2,875	**$13**	**0.4%**	$1,019	$1,002	**98.3%**	$1,018	$1,002	98.4%	$3,916	$3,871	**98.9%**	0.349	0.348	$1,296	$1,276	98.4%	1.01	1.01	1.23	1.23
Logg	34.5	3.15	$2,298	$2,321	**$23**	**1.0%**	$2,295	**-$3**	**-0.1%**	$710	$701	**98.7%**	$710	$701	98.7%	$2,686	$2,647	**98.6%**	0.306	0.305	$896	$884	98.6%	0.91	0.91	1.35	1.34
Logg	25.25	2.45	$5,166	$5,282	**$116**	**2.2%**	$5,205	**$40**	**0.8%**	$1,943	$1,909	**98.2%**	$1,940	$1,908	98.4%	$7,331	$7,209	**98.3%**	0.367	0.367	$2,420	$2,397	99.1%	1.10	1.10	1.89	1.88
Logg	34.7	3.07	$4,393	$4,430	**$37**	**0.8%**	$4,378	**-$15**	**-0.3%**	$1,379	$1,360	**98.6%**	$1,379	$1,360	98.7%	$5,159	$5,082	**98.5%**	0.311	0.311	$1,725	$1,693	98.1%	0.98	0.97	1.45	1.44
TLogg	23.5	2.65	$668	$792	**$123**	**18.5%**	$689	**$21**	**3.1%**	$655	$490	**74.8%**	$643	$489	76.1%	$2,039	$1,640	**80.4%**	0.812	0.710	$523	$446	85.3%	5.72	4.15	65.49	35.78
TLogg	33	3.3	$593	$646	**$52**	**8.8%**	$604	**$11**	**1.8%**	$333	$298	**89.5%**	$329	$298	90.5%	$1,301	$1,181	**90.7%**	0.509	0.493	$354	$328	92.6%	1.83	1.76	4.97	4.58
TLogg	24.5	2.5	$3,080	$3,491	**$411**	**13.3%**	$3,129	**$49**	**1.6%**	$2,343	$1,968	**84.0%**	$2,307	$1,968	85.3%	$8,199	$7,028	**85.7%**	0.661	0.629	$2,302	$2,020	87.7%	2.34	2.18	8.77	7.60
TLogg	34.5	3.15	$2,590	$2,752	**$162**	**6.3%**	$2,605	**$15**	**0.6%**	$1,233	$1,136	**92.1%**	$1,222	$1,136	92.9%	$4,645	$4,350	**93.7%**	0.444	0.436	$1,454	$1,356	93.3%	1.55	1.52	4.21	4.06
TLogg	24.75	2.45	$5,225	$5,914	**$689**	**13.2%**	$5,327	**$102**	**2.0%**	$3,953	$3,361	**85.0%**	$3,892	$3,360	86.3%	$15,824	$13,649	**86.3%**	0.658	0.631	$3,985	$3,505	88.0%	2.22	2.10	7.50	6.70
TLogg	34.6	3.07	$4,614	$4,875	**$261**	**5.7%**	$4,622	**$8**	**0.2%**	$2,151	$1,991	**92.6%**	$2,135	$1,991	93.3%	$8,274	$7,695	**93.0%**	0.438	0.431	$2,538	$2,382	93.9%	1.37	1.35	2.53	2.44
	ξ	**θ**																									
GPD	0.8	35,000	$1,164	$1,380	**$216**	**18.6%**	$1,175	**$11**	**1.0%**	$978	$792	**81.0%**	$954	$792	83.0%	$3,443	$2,892	**84.0%**	0.691	0.674	$978	$824	84.3%	2.32	2.26	8.70	8.29
GPD	0.95	7,500	$1,402	$1,737	**$335**	**23.9%**	$1,425	**$22**	**1.6%**	$1,412	$1,094	**77.5%**	$1,372	$1,094	79.8%	$4,654	$3,740	**80.4%**	0.789	0.768	$1,330	$1,076	80.9%	2.70	2.62	11.94	11.35
GPD	0.875	47,500	$3,728	$4,522	**$794**	**21.3%**	$3,780	**$52**	**1.4%**	$3,440	$2,725	**79.2%**	$3,347	$2,725	81.4%	$11,683	$9,629	**82.4%**	0.740	0.721	$3,384	$2,784	82.3%	2.49	2.42	10.11	9.61
GPD	0.95	25,000	$4,674	$5,800	**$1,125**	**24.1%**	$4,756	**$81**	**1.7%**	$4,713	$3,652	**77.5%**	$4,576	$3,651	79.8%	$15,512	$12,466	**80.4%**	0.789	0.768	$4,440	$3,600	81.1%	2.69	2.61	11.86	11.28
GPD	0.925	50,000	$6,994	$8,588	**$1,594**	**22.8%**	$7,088	**$94**	**1.3%**	$6,812	$5,320	**78.1%**	$6,623	$5,320	80.3%	$22,671	$18,364	**81.0%**	0.771	0.751	$6,545	$5,336	81.5%	2.64	2.56	11.46	10.89
GPD	0.99	27,500	$8,195	$10,265	**$2,070**	**25.3%**	$8,329	**$134**	**1.6%**	$8,610	$6,596	**76.6%**	$8,357	$6,594	78.9%	$27,783	$22,204	**79.9%**	0.814	0.792	$7,991	$6,364	79.6%	2.82	2.74	13.17	12.52
TGPD	0.775	33,500	$1,034	$1,231	**$197**	**19.1%**	$1,046	**$12**	**1.2%**	$874	$703	**80.4%**	$851	$703	82.5%	$3,018	$2,491	**82.5%**	0.691	0.672	$890	$745	83.7%	2.52	2.44	10.98	10.33
TGPD	0.8	25,000	$1,098	$1,328	**$230**	**20.9%**	$1,121	**$23**	**2.1%**	$918	$731	**79.6%**	$889	$730	82.2%	$3,423	$2,821	**82.4%**	0.669	0.651	$959	$795	82.9%	2.01	1.93	7.04	6.46
TGPD	0.8675	50,000	$4,226	$5,102	**$876**	**20.7%**	$4,236	**$10**	**0.2%**	$3,687	$2,896	**78.5%**	$3,581	$2,896	80.9%	$13,519	$11,043	**81.7%**	0.702	0.684	$3,772	$3,064	81.2%	2.01	1.95	5.89	5.55
TGPD	0.91	31,000	$4,716	$5,810	**$1,094**	**23.2%**	$4,767	**$51**	**1.1%**	$4,275	$3,304	**77.3%**	$4,132	$3,304	80.0%	$14,834	$11,848	**79.9%**	0.711	0.693	$4,582	$3,694	80.6%	1.94	1.87	6.81	6.31
TGPD	0.92	47,500	$7,486	$9,172	**$1,686**	**22.5%**	$7,504	**$19**	**0.2%**	$7,190	$5,548	**77.2%**	$6,989	$5,548	79.4%	$24,710	$19,648	**79.5%**	0.762	0.739	$6,800	$5,470	80.4%	2.61	2.52	12.06	11.26
TGPD	0.95	35,000	$8,322	$10,408	**$2,086**	**25.1%**	$8,441	**$119**	**1.4%**	$8,429	$6,431	**76.3%**	$8,166	$6,430	78.7%	$30,104	$23,589	**78.4%**	0.785	0.762	$8,365	$6,690	80.0%	2.23	2.16	6.92	6.51

*NOTE: #simulations = 1,000; λ = 100 for 10 years so n ~ 1,000; α=0.9997

TABLE F9a

RCE vs. LDA for Regulatory Capital Estimation Under 5% Right-Tail Contamination ($m, λ=25)*

Severity Dist.	Parm1	Parm2	True RCap	Mean MLE RCap	MLE Bias	MLE Bias%	Mean RCE RCap	RCE Bias	RCE Bias%	RMSE MLE RCap	RMSE RCE RCap	RMSE RCap RCE/MLE	StdDev MLE RCap	StdDev RCE RCap	StdDev RCap RCE/MLE	95%CIs MLE RCap	95%CIs RCE RCap	95%CIs RCap RCE/MLE	CV MLE	CV RCE	IQR MLE RCap	IQR RCE RCap	IQR RCap RCE/MLE	Skew MLE RCap	Skew RCE RCap	Kurtosis MLE RCap	Kurtosis RCE RCap
	μ	σ																									
LogN	10	2	$63	$91	$28	44.6%	$80	$17	26.7%	$63	$48	76.5%	$56	$45	80.1%	$203	$169	83.3%	0.620	0.567	$57	$49	85.6%	2.31	2.01	8.95	6.91
LogN	7.7	2.55	$53	$62	$9	17.1%	$56	$3	6.5%	$34	$29	86.9%	$32	$29	89.6%	$122	$111	91.2%	0.523	0.515	$36	$32	89.8%	2.08	2.04	11.87	11.41
LogN	10.4	2.5	$649	$755	$106	16.4%	$689	$41	6.3%	$400	$349	87.3%	$386	$347	89.9%	$1,452	$1,330	91.6%	0.511	0.503	$427	$385	90.2%	2.01	1.97	11.06	10.64
LogN	9.27	2.77	$603	$724	$121	20.1%	$648	$45	7.5%	$436	$371	85.2%	$418	$368	88.0%	$1,554	$1,395	89.8%	0.578	0.568	$446	$394	88.2%	2.43	2.37	16.15	15.40
LogN	10.75	2.7	$2,012	$2,396	$384	19.1%	$2,157	$145	7.2%	$1,397	$1,198	85.8%	$1,343	$1,189	88.5%	$5,008	$4,518	90.2%	0.560	0.551	$1,447	$1,285	88.7%	2.32	2.26	14.66	14.01
LogN	9.63	2.97	$1,893	$2,330	$437	23.1%	$2,055	$162	8.5%	$1,532	$1,281	83.6%	$1,469	$1,271	86.5%	$5,393	$4,767	88.4%	0.630	0.618	$1,518	$1,316	86.7%	2.80	2.72	21.20	20.07
TLogN	10.2	1.95	$76	$91	$15	19.2%	$80	$3	4.4%	$58	$45	77.8%	$56	$45	80.1%	$203	$169	83.3%	0.620	0.567	$57	$49	85.6%	2.31	2.01	8.95	6.91
TLogN	9	2.2	$76	$103	$27	35.9%	$79	$4	4.9%	$102	$59	57.8%	$98	$59	59.9%	$360	$224	62.3%	0.953	0.739	$68	$51	75.6%	3.99	2.75	23.67	11.16
TLogN	10.7	2.385	$670	$882	$212	31.7%	$728	$58	8.7%	$719	$513	71.4%	$687	$510	74.2%	$2,476	$1,865	75.3%	0.779	0.700	$616	$488	79.2%	3.16	2.46	20.95	11.69
TLogN	9.4	2.65	$643	$970	$327	50.9%	$675	$33	5.1%	$1,192	$581	48.7%	$1,146	$580	50.6%	$3,759	$2,123	56.5%	1.182	0.859	$709	$494	69.6%	5.23	2.83	45.49	12.85
TLogN	11	2.6	$2,085	$2,965	$880	42.2%	$2,338	$253	12.1%	$3,229	$2,085	64.6%	$3,107	$2,069	66.6%	$9,520	$6,968	73.2%	1.048	0.885	$2,076	$1,614	77.8%	4.91	3.75	37.57	22.17
TLogN	10	2.8	$1,956	$2,897	$941	48.1%	$2,058	$103	5.3%	$3,457	$1,856	53.7%	$3,326	$1,853	55.7%	$10,530	$6,562	62.3%	1.148	0.900	$2,262	$1,579	69.8%	4.99	3.30	40.71	19.17
	a	b																									
Logg	24	2.65	$85	$102	$17	19.8%	$92	$6	7.6%	$69	$59	85.4%	$67	$59	87.5%	$257	$227	88.2%	0.658	0.641	$67	$60	88.9%	2.61	2.53	11.90	11.23
Logg	33	3.3	$100	$114	$14	14.2%	$104	$5	4.9%	$62	$54	87.9%	$60	$54	89.9%	$221	$198	89.6%	0.531	0.520	$64	$58	91.1%	1.86	1.79	5.58	5.09
Logg	25	2.5	$444	$535	$92	20.7%	$474	$30	6.8%	$362	$303	83.8%	$350	$302	86.1%	$1,356	$1,159	85.4%	0.654	0.637	$346	$302	87.4%	2.01	1.92	6.22	5.54
Logg	34.5	3.15	$448	$516	$68	15.2%	$469	$21	4.7%	$300	$261	87.0%	$292	$260	89.0%	$1,113	$992	89.2%	0.567	0.555	$313	$281	90.0%	1.88	1.84	5.15	4.99
Logg	25.25	2.45	$766	$933	$167	21.7%	$821	$55	7.2%	$697	$580	83.3%	$677	$578	85.4%	$2,340	$2,027	86.6%	0.725	0.704	$607	$525	86.4%	3.11	2.98	15.62	14.47
Logg	34.7	3.07	$818	$947	$129	15.8%	$857	$39	4.8%	$572	$495	86.5%	$557	$493	88.5%	$2,107	$1,862	88.4%	0.588	0.575	$601	$536	89.1%	2.06	2.00	7.22	6.80
TLogg	23.5	2.65	$124	$237	$113	91.5%	$151	$28	22.3%	$906	$790	87.1%	$899	$789	87.8%	$932	$471	50.5%	3.790	5.211	$173	$95	54.6%	27.17	29.88	811.09	925.45
TLogg	33	3.3	$130	$180	$51	39.3%	$132	$2	1.9%	$202	$96	47.6%	$195	$96	49.2%	$539	$323	60.0%	1.081	0.727	$134	$95	70.5%	4.71	2.85	31.29	14.03
TLogg**	24.5	2.5	$495	$1,158	$663	133.8%	$629	$134	27.0%	$9,870	$4,828	48.9%	$9,847	$4,826	49.0%	$3,331	$1,456	43.7%	8.504	7.673	$699	$366	52.3%	31.14	31.34	979.37	988.09
TLogg	34.5	3.15	$510	$671	$162	31.8%	$560	$50	9.8%	$661	$464	70.2%	$641	$461	72.0%	$2,171	$1,620	74.6%	0.954	0.824	$499	$414	83.0%	3.86	3.03	22.38	14.12
TLogg##	24.75	2.45	$801	$1,547	$746	93.1%	$896	$95	11.8%	$2,251	$747	33.2%	$2,124	$741	34.9%	$6,347	$2,705	42.6%	1.373	0.827	$1,264	$748	59.1%	6.69	2.82	78.91	14.85
TLogg	34.6	3.07	$867	$1,123	$256	29.6%	$956	$89	10.3%	$1,030	$777	75.4%	$998	$772	77.4%	$3,409	$2,702	79.2%	0.889	0.808	$799	$664	83.1%	3.52	3.11	19.50	15.69
	ξ	θ																									
GPD	0.8	35,000	$149	$248	$99	66.5%	$160	$12	7.9%	$323	$178	55.2%	$307	$178	57.8%	$1,025	$600	58.6%	1.242	1.108	$216	$139	64.7%	3.87	3.40	21.19	16.68
GPD	0.95	7,500	$121	$228	$107	88.8%	$132	$11	9.4%	$353	$171	48.4%	$336	$170	50.7%	$1,059	$561	52.9%	1.475	1.289	$204	$121	59.2%	4.73	4.05	32.21	24.20
GPD	0.875	47,500	$391	$679	$287	73.5%	$417	$26	6.8%	$937	$487	52.0%	$891	$487	54.6%	$3,052	$1,726	56.6%	1.314	1.166	$595	$368	61.7%	4.02	3.48	22.93	17.52
GPD	0.95	25,000	$403	$742	$340	84.4%	$432	$29	7.2%	$1,105	$538	48.7%	$1,051	$538	51.2%	$3,521	$1,868	53.1%	1.416	1.245	$660	$392	59.4%	4.31	3.71	26.17	19.95
GPD	0.925	50,000	$643	$1,149	$505	78.6%	$682	$38	6.0%	$1,666	$832	49.9%	$1,588	$831	52.3%	$5,261	$2,926	55.6%	1.382	1.219	$1,042	$633	60.7%	4.29	3.69	26.10	19.77
GPD	0.99	27,500	$636	$1,202	$566	89.1%	$678	$42	6.6%	$1,859	$875	47.1%	$1,770	$874	49.4%	$5,455	$3,006	55.1%	1.473	1.289	$1,100	$630	57.2%	4.53	3.90	28.95	22.21
TGPD	0.775	33,500	$141	$218	$77	54.4%	$147	$6	4.0%	$260	$153	58.8%	$248	$153	61.5%	$912	$565	62.0%	1.139	1.040	$180	$119	65.9%	3.24	3.04	14.73	13.25
TGPD	0.8	25,000	$140	$222	$82	58.1%	$146	$6	4.2%	$296	$166	56.1%	$284	$166	58.3%	$938	$562	60.0%	1.281	1.132	$189	$125	66.1%	5.68	4.71	56.76	38.75
TGPD	0.8675	50,000	$452	$766	$314	69.5%	$484	$32	7.0%	$1,006	$548	54.5%	$956	$547	57.3%	$3,231	$1,959	60.6%	1.247	1.131	$662	$415	62.6%	3.30	2.98	14.60	11.67
TGPD	0.91	31,000	$451	$789	$338	74.9%	$482	$31	6.8%	$1,147	$599	52.3%	$1,096	$599	54.6%	$3,504	$1,957	55.9%	1.388	1.242	$655	$402	61.4%	4.22	3.81	24.90	20.60
TGPD	0.92	47,500	$698	$1,223	$526	75.4%	$741	$44	6.3%	$1,960	$1,008	51.4%	$1,888	$1,007	53.3%	$5,529	$3,130	56.6%	1.543	1.358	$993	$616	62.0%	5.96	5.19	55.30	42.24
TGPD	0.95	35,000	$717	$1,364	$647	90.3%	$802	$85	11.8%	$2,479	$1,219	49.2%	$2,393	$1,216	50.8%	$6,397	$3,594	56.2%	1.754	1.516	$1,019	$605	59.4%	7.15	6.10	79.49	59.77

*NOTE: #simulations = 1,000; λ = 25 for 10 years so n ~ 250; α=0.999

** 15% contamination used instead of 5%.

25% contamination used instead of 5%.

TABLE F9b

RCE vs. LDA for Economic Capital Estimation Under 5% Right-Tail Contamination ($m, λ = 25)*

Severity Dist.	Parm1	Parm2	True ECap	Mean MLE ECap	MLE Bias	MLE Bias%	Mean RCE ECap	RCE Bias	RCE Bias%	RMSE MLE ECap	RMSE RCE ECap	RMSE ECap RCE/MLE	StdDev MLE ECap	StdDev RCE ECap	StdDev ECap RCE/MLE	95%CIs MLE ECap	95%CIs RCE ECap	95%CIs ECap RCE/MLE	CV MLE	CV RCE	IQR MLE ECap	IQR RCE ECap	IQR ECap RCE/MLE	Skew MLE ECap	Skew RCE ECap	Kurtosis MLE ECap	Kurtosis RCE ECap
	μ	σ																									
LogN	10	2	$107	$155	**$48**	**45.2%**	$133	**$26**	**24.1%**	$120	$88	**73.6%**	$110	$85	77.0%	$391	$316	**80.9%**	0.708	0.638	$104	$87	83.3%	2.65	2.25	11.58	8.56
LogN	7.7	2.55	$107	$127	**$20**	**19.1%**	$115	**$8**	**7.8%**	$75	$64	**86.4%**	$72	$64	89.1%	$268	$239	**89.3%**	0.565	0.556	$78	$70	89.9%	2.39	2.34	15.86	15.22
LogN	10.4	2.5	$1,286	$1,523	**$237**	**18.4%**	$1,383	**$97**	**7.5%**	$872	$757	**86.8%**	$840	$751	89.4%	$3,146	$2,819	**89.6%**	0.551	0.543	$924	$833	90.2%	2.30	2.26	14.72	14.15
LogN	9.27	2.77	$1,293	$1,584	**$292**	**22.6%**	$1,410	**$117**	**9.0%**	$1,033	$874	**84.6%**	$991	$866	87.4%	$3,653	$3,201	**87.6%**	0.625	0.614	$1,039	$920	88.5%	2.83	2.75	21.87	20.83
LogN	10.75	2.7	$4,230	$5,137	**$907**	**21.4%**	$4,595	**$366**	**8.6%**	$3,241	$2,760	**85.2%**	$3,111	$2,735	87.9%	$11,523	$10,160	**88.2%**	0.606	0.595	$3,306	$2,941	88.9%	2.68	2.62	19.76	18.87
LogN	9.63	2.97	$4,303	$5,421	**$1,118**	**26.0%**	$4,743	**$440**	**10.2%**	$3,869	$3,206	**82.9%**	$3,703	$3,176	85.7%	$13,459	$11,586	**86.1%**	0.683	0.670	$3,737	$3,254	87.1%	3.29	3.19	29.05	27.46
TLogN	10.2	1.95	$126	$155	**$30**	**23.6%**	$133	**$7**	**5.6%**	$114	$85	**74.6%**	$110	$85	77.0%	$391	$316	**80.9%**	0.708	0.638	$104	$87	83.3%	2.65	2.25	11.58	8.56
TLogN	9	2.2	$133	$195	**$62**	**46.6%**	$140	**$7**	**5.5%**	$232	$116	**50.1%**	$223	$116	51.9%	$777	$429	**55.2%**	1.145	0.826	$134	$96	71.5%	4.78	2.94	34.11	12.70
TLogN	10.7	2.385	$1,267	$1,753	**$486**	**38.3%**	$1,397	**$130**	**10.3%**	$1,629	$1,096	**67.3%**	$1,555	$1,089	70.0%	$5,510	$3,940	**71.5%**	0.887	0.779	$1,292	$1,001	77.4%	3.88	2.83	32.01	15.71
TLogN	9.4	2.65	$1,297	$2,153	**$856**	**66.0%**	$1,369	**$72**	**5.5%**	$3,199	$1,303	**40.7%**	$3,083	$1,301	42.2%	$9,216	$4,795	**52.0%**	1.432	0.950	$1,591	$1,036	65.1%	6.66	3.06	72.84	14.99
TLogN	11	2.6	$4,208	$6,408	**$2,200**	**52.3%**	$4,822	**$614**	**14.6%**	$8,217	$4,863	**59.2%**	$7,916	$4,824	60.9%	$22,384	$15,736	**70.3%**	1.235	1.000	$4,589	$3,488	76.0%	5.84	4.22	51.84	27.39
TLogN	10	2.8	$4,145	$6,678	**$2,533**	**61.1%**	$4,391	**$245**	**5.9%**	$9,445	$4,384	**46.4%**	$9,099	$4,377	48.1%	$27,395	$15,340	**56.0%**	1.362	0.997	$5,303	$3,523	66.4%	6.06	3.64	57.58	23.07
	a	b																									
Logg	24	2.65	$192	$238	**$46**	**23.9%**	$211	**$18**	**9.5%**	$182	$152	**83.6%**	$176	$151	85.8%	$666	$573	**86.1%**	0.740	0.718	$168	$146	86.9%	2.95	2.85	14.86	13.93
Logg	33	3.3	$203	$238	**$35**	**17.0%**	$216	**$12**	**6.2%**	$145	$125	**86.5%**	$141	$125	88.6%	$508	$453	**89.1%**	0.593	0.579	$142	$127	89.3%	2.10	2.01	7.19	6.52
Logg	25	2.5	$1,064	$1,329	**$266**	**25.0%**	$1,154	**$91**	**8.5%**	$1,005	$821	**81.7%**	$969	$816	84.2%	$3,706	$3,130	**84.5%**	0.729	0.707	$915	$791	86.5%	2.27	2.15	7.97	7.03
Logg	34.5	3.15	$960	$1,135	**$175**	**18.2%**	$1,017	**$57**	**6.0%**	$735	$629	**85.5%**	$714	$626	87.6%	$2,697	$2,386	**88.5%**	0.630	0.615	$738	$647	87.7%	2.06	2.01	6.18	5.94
Logg	25.25	2.45	$1,877	$2,372	**$495**	**26.3%**	$2,046	**$169**	**9.0%**	$2,004	$1,626	**81.1%**	$1,942	$1,617	83.3%	$6,583	$5,586	**84.9%**	0.819	0.790	$1,635	$1,387	84.8%	3.52	3.36	19.42	17.86
Logg	34.7	3.07	$1,794	$2,135	**$341**	**19.0%**	$1,905	**$110**	**6.2%**	$1,439	$1,221	**84.8%**	$1,398	$1,216	86.9%	$5,220	$4,546	**87.1%**	0.655	0.638	$1,421	$1,273	89.6%	2.31	2.24	9.01	8.40
TLogg	23.5	2.65	$271	$703	**$432**	**159.7%**	**$394**	**$124**	**45.7%**	$5,152	$4,436	**86.1%**	$5,133	$4,434	86.4%	$2,834	$1,065	**37.6%**	7.302	11.241	$444	$190	42.8%	30.11	31.23	935.40	983.28
TLogg	33	3.3	$261	$400	**$139**	**53.2%**	**$248**	**-$13**	**-5.0%**	$554	$189	**34.1%**	$536	$188	35.1%	$1,337	$630	**47.1%**	1.340	0.759	$302	$186	61.6%	5.57	3.07	42.29	16.94
TLogg**	24.5	2.5	$1,164	$4,748	**$3,584**	**307.9%**	$1,978	**$814**	**70.0%**	$78,844	$33,228	**42.1%**	$78,762	$33,218	42.2%	$10,042	$2,896	**28.8%**	16.587	16.790	$1,912	$711	37.2%	31.54	31.60	996.57	998.89
TLogg	34.5	3.15	$1,086	$1,537	**$451**	**41.5%**	**$1,205**	**$119**	**10.9%**	$1,796	$1,105	**61.5%**	$1,739	$1,099	63.2%	$5,537	$3,848	**69.5%**	1.131	0.912	$1,194	$937	78.5%	4.52	3.21	29.62	15.57
TLogg##	24.75	2.45	$1,928	$4,297	**$2,369**	**122.9%**	$1,863	**-$65**	**-3.4%**	$7,864	$1,579	**20.1%**	$7,499	$1,578	21.0%	$19,566	$5,575	**28.5%**	1.745	0.847	$3,550	$1,562	44.0%	9.02	3.07	132.89	18.36
TLogg	34.6	3.07	$1,892	$2,607	**$715**	**37.8%**	**$2,121**	**$229**	**12.1%**	$2,793	$1,942	**69.5%**	$2,700	$1,928	71.4%	$9,079	$6,624	**73.0%**	1.036	0.909	$1,936	$1,561	80.6%	4.05	3.44	25.19	18.98
	ξ	θ																									
GPD	0.8	35,000	$382	$745	**$362**	**94.7%**	$421	**$38**	**10.0%**	$1,189	$559	**47.0%**	$1,133	$558	49.3%	$3,487	$1,792	**51.4%**	1.522	1.327	$653	$376	57.5%	4.63	3.99	29.42	22.45
GPD	0.95	7,500	$375	$855	**$480**	**127.8%**	$419	**$43**	**11.5%**	$1,627	$650	**40.0%**	$1,555	$649	41.7%	$4,591	$2,027	**44.1%**	1.818	1.549	$734	$383	52.2%	5.64	4.78	43.61	32.59
GPD	0.875	47,500	$1,106	$2,268	**$1,162**	**105.0%**	$1,199	**$92**	**8.4%**	$3,836	$1,675	**43.7%**	$3,655	$1,672	45.8%	$11,760	$5,646	**48.0%**	1.612	1.395	$1,965	$1,076	54.8%	4.88	4.16	32.90	24.65
GPD	0.95	25,000	$1,251	$2,767	**$1,516**	**121.2%**	$1,361	**$110**	**8.8%**	$5,045	$2,033	**40.3%**	$4,812	$2,030	42.2%	$14,885	$6,672	**44.8%**	1.739	1.492	$2,385	$1,242	52.1%	5.20	4.43	36.82	27.73
GPD	0.925	50,000	$1,938	$4,121	**$2,183**	**112.7%**	$2,080	**$142**	**7.3%**	$7,342	$3,043	**41.4%**	$7,010	$3,039	43.4%	$21,907	$10,180	**46.5%**	1.701	1.461	$3,650	$1,930	52.9%	5.20	4.42	36.98	27.76
GPD	0.99	27,500	$2,076	$4,743	**$2,667**	**128.5%**	$2,243	**$167**	**8.0%**	$8,997	$3,470	**38.6%**	$8,592	$3,466	40.3%	$24,631	$11,151	**45.3%**	1.811	1.545	$4,195	$2,119	50.5%	5.45	4.64	40.30	30.48
TGPD	0.775	33,500	$351	$623	**$272**	**77.3%**	$370	**$19**	**5.3%**	$900	$459	**51.0%**	$858	$459	53.5%	$3,044	$1,623	**53.3%**	1.378	1.241	$524	$315	60.1%	3.83	3.56	20.66	18.32
TGPD	0.8	25,000	$361	$662	**$301**	**83.4%**	$381	**$20**	**5.5%**	$1,113	$528	**47.4%**	$1,072	$528	49.2%	$3,265	$1,682	**51.5%**	1.620	1.386	$573	$336	58.5%	7.48	5.90	93.28	58.49
TGPD	0.8675	50,000	$1,267	$2,517	**$1,250**	**98.7%**	$1,376	**$110**	**8.7%**	$3,962	$1,837	**46.4%**	$3,759	$1,834	48.8%	$12,050	$6,352	**52.7%**	1.494	1.332	$2,143	$1,201	56.0%	3.86	3.42	20.09	15.44
TGPD	0.91	31,000	$1,334	$2,766	**$1,433**	**107.4%**	$1,445	**$111**	**8.3%**	$4,904	$2,154	**43.9%**	$4,690	$2,152	45.9%	$13,991	$6,589	**47.1%**	1.695	1.489	$2,259	$1,224	54.2%	4.99	4.49	33.95	28.04
TGPD	0.92	47,500	$2,088	$4,377	**$2,289**	**109.6%**	$2,264	**$176**	**8.4%**	$8,814	$3,785	**42.9%**	$8,511	$3,781	44.4%	$22,898	$10,671	**46.6%**	1.944	1.670	$3,446	$1,887	54.8%	7.34	6.27	81.48	59.87
TGPD	0.95	35,000	$2,227	$5,195	**$2,967**	**133.2%**	$2,574	**$347**	**15.6%**	$12,102	$4,880	**40.3%**	$11,733	$4,868	41.5%	$27,632	$13,122	**47.5%**	2.258	1.891	$3,692	$1,894	51.3%	8.99	7.58	119.45	88.69

*NOTE: #simulations = 1,000; λ = 25 for 10 years so n ~ 250; α=0.9997

** 15% contamination used instead of 5%.

25% contamination used instead of 5%.

TABLE F10a

RCE vs. LDA for Regulatory Capital Estimation Under 5% Left-Tail Contamination ($m, λ=25)*

Severity Dist.	Parm1	Parm2	True RCap	Mean MLE RCap	MLE Bias	MLE Bias%	Mean RCE RCap	RCE Bias	RCE Bias%	RMSE MLE RCap	RMSE RCE RCap	RMSE RCap RCE/MLE	StdDev MLE RCap	StdDev RCE RCap	StdDev RCap RCE/MLE	95%CIs MLE RCap	95%CIs RCE RCap	95%CIs RCap RCE/MLE	CV MLE	CV RCE	IQR MLE RCap	IQR RCE RCap	IQR RCap RCE/MLE	Skew MLE RCap	Skew RCE RCap	Kurtosis MLE RCap	Kurtosis RCE RCap
	μ	σ																									
LogN	10	2	$63	$64	$1	2.1%	$60	-$2	-3.8%	$25	$23	93.5%	$25	$23	93.2%	$93	$87	93.7%	0.388	0.384	$30	$28	93.3%	1.46	1.44	6.13	5.98
LogN	7.7	2.55	$53	$56	$3	5.5%	$51	-$2	-3.9%	$29	$26	89.6%	$29	$26	89.7%	$106	$96	90.2%	0.521	0.513	$33	$30	90.5%	2.23	2.18	14.22	13.63
LogN	10.4	2.5	$649	$682	$33	5.1%	$623	-$25	-3.9%	$348	$313	89.9%	$347	$312	90.0%	$1,266	$1,147	90.6%	0.509	0.501	$391	$355	90.8%	2.15	2.10	13.21	12.69
LogN	9.27	2.77	$603	$646	$43	7.2%	$579	-$23	-3.9%	$375	$329	87.8%	$372	$328	88.2%	$1,337	$1,186	88.7%	0.577	0.567	$403	$359	89.1%	2.63	2.56	19.50	18.56
LogN	10.75	2.7	$2,012	$2,145	$133	6.6%	$1,933	-$78	-3.9%	$1,206	$1,065	88.4%	$1,198	$1,062	88.7%	$4,322	$3,855	89.2%	0.559	0.549	$1,312	$1,174	89.5%	2.49	2.43	17.65	16.84
LogN	9.63	2.97	$1,893	$2,062	$169	8.9%	$1,821	-$72	-3.8%	$1,309	$1,127	86.1%	$1,298	$1,125	86.7%	$4,592	$4,009	87.3%	0.630	0.618	$1,358	$1,192	87.8%	3.05	2.96	25.77	24.35
TLogN	10.2	1.95	$76	$85	$9	11.6%	$75	-$2	-2.0%	$53	$42	79.4%	$52	$42	80.5%	$189	$153	80.8%	0.612	0.561	$53	$45	84.5%	2.62	2.28	14.74	11.21
TLogN	9	2.2	$76	$98	$22	28.8%	$74	-$1	-1.9%	$101	$53	52.4%	$99	$53	53.6%	$276	$178	64.7%	1.015	0.715	$69	$51	73.9%	5.32	2.87	43.35	14.59
TLogN	10.7	2.385	$670	$826	$157	23.4%	$680	$10	1.5%	$738	$524	71.0%	$721	$524	72.6%	$2,304	$1,788	77.6%	0.873	0.771	$580	$460	79.3%	3.89	3.12	25.93	16.51
TLogN	9.4	2.65	$643	$857	$214	33.4%	$612	-$31	-4.8%	$926	$520	56.2%	$900	$519	57.6%	$3,166	$1,899	60.0%	1.051	0.848	$645	$452	70.1%	3.89	3.00	25.33	15.87
TLogN	11	2.6	$2,085	$2,737	$652	31.3%	$2,154	$69	3.3%	$3,254	$2,040	62.7%	$3,188	$2,039	64.0%	$9,160	$6,699	73.1%	1.165	0.947	$2,051	$1,597	77.9%	7.47	5.09	92.10	45.06
TLogN	10	2.8	$1,956	$2,652	$697	35.6%	$1,889	-$67	-3.4%	$3,317	$1,831	55.2%	$3,242	$1,830	56.4%	$9,669	$6,401	66.2%	1.222	0.969	$1,980	$1,425	72.0%	5.70	3.96	52.65	27.05
	a	b																									
Logg	24	2.65	$85	$97	$11	13.0%	$87	$1	1.5%	$65	$56	86.1%	$64	$56	87.3%	$226	$200	88.4%	0.664	0.645	$59	$52	89.0%	2.86	2.75	13.53	12.63
Logg	33	3.3	$100	$109	$9	9.2%	$100	$0	0.3%	$57	$51	88.8%	$56	$51	90.0%	$216	$193	89.3%	0.519	0.509	$59	$54	90.8%	1.65	1.60	3.92	3.70
Logg	25	2.5	$444	$505	$62	13.9%	$447	$4	0.9%	$336	$285	84.8%	$330	$285	86.2%	$1,261	$1,092	86.6%	0.653	0.636	$323	$284	87.8%	1.99	1.91	5.85	5.28
Logg	34.5	3.15	$448	$493	$45	10.0%	$448	$0	0.0%	$291	$257	88.2%	$288	$257	89.2%	$1,057	$942	89.1%	0.584	0.573	$296	$265	89.7%	2.42	2.40	11.32	11.29
Logg	25.25	2.45	$766	$876	$109	14.3%	$771	$5	0.7%	$630	$531	84.3%	$620	$531	85.6%	$2,250	$1,961	87.2%	0.709	0.689	$570	$496	87.1%	3.02	2.92	15.40	14.53
Logg	34.7	3.07	$818	$902	$85	10.3%	$817	-$1	-0.1%	$541	$473	87.5%	$535	$473	88.5%	$2,049	$1,803	88.0%	0.592	0.579	$560	$492	87.9%	2.11	2.07	7.12	6.84
TLogg	23.5	2.65	$124	$222	$98	79.5%	$142	$18	14.7%	$591	$490	83.0%	$582	$490	84.2%	$959	$474	49.4%	2.619	3.450	$143	$83	58.3%	17.83	23.52	411.12	641.75
TLogg	33	3.3	$130	$168	$39	29.9%	$125	-$5	-3.8%	$180	$93	51.6%	$176	$93	52.8%	$519	$304	58.5%	1.044	0.744	$132	$91	68.9%	5.87	3.92	64.96	36.53
TLogg	24.5	2.5	$495	$806	$311	62.8%	$425	-$70	-14.2%	$1,380	$447	32.4%	$1,344	$441	32.8%	$3,194	$1,238	38.8%	1.667	1.038	$636	$319	50.2%	8.06	5.89	96.75	60.93
TLogg	34.5	3.15	$510	$630	$120	23.6%	$525	$15	3.0%	$641	$446	69.5%	$630	$446	70.7%	$1,972	$1,516	76.9%	1.000	0.849	$455	$375	82.3%	5.95	5.06	59.69	49.38
TLogg##	24.75	2.45	$801	$1,244	$443	55.3%	$684	-$117	-14.6%	$2,250	$653	29.0%	$2,206	$643	29.1%	$5,169	$1,858	36.0%	1.774	0.939	$964	$503	52.2%	12.16	5.60	227.64	58.78
TLogg	34.6	3.07	$867	$1,074	$208	24.0%	$912	$45	5.2%	$920	$687	74.7%	$896	$685	76.5%	$3,141	$2,470	78.6%	0.834	0.751	$824	$678	82.3%	2.94	2.47	13.34	9.91
	ξ	θ																									
GPD	0.8	35,000	$149	$216	$67	45.0%	$141	-$8	-5.2%	$275	$156	56.6%	$267	$156	58.3%	$886	$545	61.5%	1.237	1.103	$177	$113	63.6%	3.92	3.47	22.03	17.52
GPD	0.95	7,500	$121	$195	$75	61.7%	$114	-$6	-5.3%	$296	$147	49.7%	$287	$147	51.3%	$891	$489	54.8%	1.468	1.286	$159	$95	59.6%	4.67	4.04	30.71	23.59
GPD	0.875	47,500	$391	$590	$199	51.0%	$367	-$24	-6.2%	$804	$430	53.5%	$779	$429	55.2%	$2,605	$1,496	57.4%	1.319	1.171	$494	$303	61.4%	4.14	3.62	24.70	19.15
GPD	0.95	25,000	$403	$650	$247	61.5%	$381	-$22	-5.4%	$987	$491	49.7%	$955	$490	51.3%	$2,972	$1,628	54.8%	1.469	1.287	$533	$317	59.5%	4.68	4.05	30.79	23.68
GPD	0.925	50,000	$643	$993	$350	54.4%	$595	-$48	-7.5%	$1,422	$730	51.3%	$1,378	$728	52.8%	$4,498	$2,519	56.0%	1.387	1.223	$842	$506	60.1%	4.41	3.81	27.97	21.39
GPD	0.99	27,500	$636	$1,036	$401	63.0%	$590	-$45	-7.1%	$1,580	$764	48.4%	$1,529	$763	49.9%	$4,825	$2,569	53.3%	1.475	1.292	$857	$493	57.5%	4.62	4.00	30.51	23.59
TGPD	0.775	33,500	$141	$206	$65	45.8%	$138	-$3	-2.0%	$272	$159	58.4%	$264	$159	60.2%	$894	$570	63.8%	1.285	1.149	$173	$115	66.6%	4.35	3.78	29.45	22.10
TGPD**	0.8	25,000	$140	$189	$49	34.9%	$126	-$14	-10.1%	$236	$140	59.3%	$231	$139	60.3%	$742	$459	61.8%	1.222	1.105	$155	$101	65.2%	4.06	3.77	23.69	20.84
TGPD	0.8675	50,000	$452	$676	$224	49.6%	$429	-$23	-5.1%	$1,014	$558	55.0%	$989	$557	56.4%	$3,353	$1,938	57.8%	1.462	1.299	$537	$337	62.8%	5.69	5.00	51.06	39.99
TGPD	0.91	31,000	$451	$678	$227	50.3%	$419	-$32	-7.1%	$936	$502	53.7%	$908	$501	55.2%	$3,022	$1,785	59.1%	1.338	1.196	$548	$338	61.6%	3.97	3.62	20.76	17.57
TGPD	0.92	47,500	$698	$1,129	$432	61.9%	$686	-$12	-1.7%	$1,943	$1,003	51.6%	$1,894	$1,003	53.0%	$5,193	$2,987	57.5%	1.677	1.462	$913	$555	60.8%	7.77	6.69	101.10	78.70
TGPD	0.95	35,000	$717	$1,128	$411	57.3%	$675	-$42	-5.9%	$1,795	$916	51.1%	$1,747	$915	52.4%	$5,160	$2,905	56.3%	1.549	1.357	$919	$545	59.3%	5.98	5.12	55.57	41.61

*NOTE: #simulations = 1,000; λ = 25 for 10 years so n ~ 250; α=0.999

** 10% contamination instead of 5%.

25% contamination used instead of 5%.

TABLE F10b

RCE vs. LDA for Economic Capital Estimation Under 5% Left-Tail Contamination ($m, λ = 25)*

Severity Dist.	Parm1	Parm2	True ECap	Mean MLE ECap	MLE Bias	MLE Bias%	Mean RCE ECap	RCE Bias	RCE Bias%	RMSE MLE ECap	RMSE RCE ECap	RMSE ECap RCE/MLE	StdDev MLE ECap	StdDev RCE ECap	StdDev ECap RCE/MLE	95%CIs MLE ECap	95%CIs RCE ECap	95%CIs ECap RCE/MLE	CV MLE	CV RCE	IQR MLE ECap	IQR RCE ECap	IQR ECap RCE/MLE	Skew MLE ECap	Skew RCE ECap	Kurtosis MLE ECap	Kurtosis RCE ECap
	μ	σ																									
LogN	10	2	$107	$110	$3	2.8%	$103	-$4	-3.5%	$46	$43	93.0%	$46	$43	92.8%	$171	$158	92.3%	0.420	0.415	$55	$51	92.6%	1.65	1.63	7.97	7.78
LogN	7.7	2.55	$107	$114	$7	6.7%	$103	-$4	-3.3%	$64	$57	88.8%	$64	$57	89.2%	$230	$204	88.4%	0.563	0.554	$70	$62	88.3%	2.58	2.52	19.12	18.32
LogN	10.4	2.5	$1,286	$1,367	$81	6.3%	$1,243	-$43	-3.3%	$755	$674	89.2%	$751	$672	89.5%	$2,711	$2,406	88.7%	0.550	0.541	$833	$739	88.7%	2.48	2.43	17.71	17.00
LogN	9.27	2.77	$1,293	$1,405	$112	8.7%	$1,252	-$41	-3.1%	$885	$769	86.9%	$878	$768	87.5%	$3,103	$2,689	86.7%	0.625	0.613	$929	$806	86.8%	3.08	2.99	26.58	25.27
LogN	10.75	2.7	$4,230	$4,570	$340	8.0%	$4,094	-$135	-3.2%	$2,784	$2,436	87.5%	$2,763	$2,433	88.0%	$9,824	$8,569	87.2%	0.605	0.594	$2,963	$2,586	87.3%	2.91	2.84	23.96	22.83
LogN	9.63	2.97	$4,303	$4,765	$462	10.7%	$4,177	-$126	-2.9%	$3,290	$2,800	85.1%	$3,257	$2,797	85.9%	$11,312	$9,624	85.1%	0.684	0.670	$3,314	$2,827	85.3%	3.62	3.50	35.53	33.52
TLogN	10.2	1.95	$126	$145	$19	15.3%	$124	-$2	-1.3%	$103	$79	76.1%	$101	$79	77.4%	$358	$286	79.8%	0.701	0.634	$97	$80	82.9%	3.08	2.61	20.27	14.76
TLogN	9	2.2	$133	$185	$52	39.3%	$130	-$3	-2.2%	$239	$102	42.5%	$233	$102	43.5%	$599	$348	58.1%	1.259	0.780	$139	$97	69.6%	6.44	2.94	59.90	15.85
TLogN	10.7	2.385	$1,267	$1,644	$377	29.7%	$1,303	$36	2.9%	$1,702	$1,131	66.5%	$1,659	$1,131	68.1%	$5,182	$3,732	72.0%	1.009	0.867	$1,213	$930	76.7%	4.64	3.54	36.27	20.95
TLogN	9.4	2.65	$1,297	$1,869	$572	44.1%	$1,232	-$65	-5.0%	$2,354	$1,158	49.2%	$2,283	$1,156	50.6%	$7,926	$4,178	52.7%	1.222	0.939	$1,442	$959	66.5%	4.46	3.29	32.62	19.13
TLogN	11	2.6	$4,208	$5,910	$1,702	40.4%	$4,433	$224	5.3%	$8,521	$4,806	56.4%	$8,349	$4,800	57.5%	$20,903	$15,066	72.1%	1.413	1.083	$4,522	$3,443	76.1%	9.22	5.89	132.02	58.19
TLogN	10	2.8	$4,145	$6,099	$1,954	47.1%	$4,021	-$124	-3.0%	$9,060	$4,343	47.9%	$8,846	$4,342	49.1%	$24,136	$14,824	61.4%	1.450	1.080	$4,625	$3,102	67.1%	6.78	4.38	70.89	32.61
	a	b																									
Logg	24	2.65	$192	$224	$32	16.5%	$198	$6	2.9%	$171	$144	84.1%	$168	$144	85.6%	$569	$495	87.0%	0.750	0.726	$144	$128	88.9%	3.22	3.10	16.65	15.51
Logg	33	3.3	$203	$227	$24	11.7%	$206	$3	1.3%	$133	$116	87.3%	$131	$116	88.7%	$494	$442	89.4%	0.578	0.565	$132	$119	90.5%	1.83	1.77	4.88	4.57
Logg	25	2.5	$1,064	$1,252	$188	17.7%	$1,088	$24	2.3%	$931	$768	82.6%	$911	$768	84.3%	$3,405	$2,898	85.1%	0.728	0.706	$852	$741	87.0%	2.23	2.13	7.44	6.65
Logg	34.5	3.15	$960	$1,082	$122	12.7%	$971	$11	1.1%	$716	$620	86.5%	$706	$620	87.8%	$2,570	$2,248	87.5%	0.652	0.638	$694	$610	87.8%	2.72	2.68	14.04	13.89
Logg	25.25	2.45	$1,877	$2,218	$341	18.2%	$1,916	$39	2.1%	$1,802	$1,479	82.1%	$1,770	$1,479	83.6%	$6,253	$5,258	84.1%	0.798	0.772	$1,519	$1,295	85.3%	3.43	3.30	19.41	18.17
Logg	34.7	3.07	$1,794	$2,031	$237	13.2%	$1,812	$18	1.0%	$1,360	$1,166	85.7%	$1,339	$1,165	87.0%	$5,075	$4,428	87.2%	0.659	0.643	$1,319	$1,173	88.9%	2.34	2.28	8.65	8.23
TLogg	23.5	2.65	$271	$630	$359	132.8%	$344	$73	27.1%	$2,787	$2,374	85.2%	$2,764	$2,373	85.9%	$2,925	$1,142	39.0%	4.387	6.897	$360	$171	47.7%	22.70	27.83	599.55	828.72
TLogg	33	3.3	$261	$371	$110	41.9%	$234	-$27	-10.5%	$497	$187	37.7%	$484	$185	38.3%	$1,327	$579	43.7%	1.307	0.793	$296	$177	59.9%	7.68	4.67	104.18	52.17
TLogg	24.5	2.5	$1,164	$2,232	$1,068	91.8%	$824	-$340	-29.2%	$4,992	$1,085	21.7%	$4,877	$1,030	21.1%	$9,990	$2,694	27.0%	2.185	1.250	$1,677	$618	36.8%	9.71	9.50	129.78	154.87
TLogg	34.5	3.15	$1,086	$1,436	$350	32.2%	$1,124	$37	3.4%	$1,776	$1,073	60.4%	$1,741	$1,072	61.6%	$5,098	$3,455	67.8%	1.212	0.954	$1,097	$846	77.1%	6.99	5.76	76.69	62.48
TLogg##	24.75	2.45	$1,928	$3,467	$1,539	79.8%	$1,381	-$547	-28.4%	$8,408	$1,462	17.4%	$8,266	$1,356	16.4%	$15,646	$4,054	25.9%	2.384	0.982	$2,651	$1,017	38.4%	15.16	5.53	323.75	53.12
TLogg	34.6	3.07	$1,892	$2,488	$596	31.5%	$2,016	$124	6.6%	$2,483	$1,690	68.1%	$2,410	$1,685	69.9%	$8,178	$6,061	74.1%	0.969	0.836	$1,977	$1,572	79.5%	3.46	2.73	17.98	12.13
	ξ	θ																									
GPD	0.8	35,000	$382	$637	$254	66.5%	$364	-$19	-4.9%	$1,001	$483	48.2%	$968	$482	49.8%	$3,064	$1,646	53.7%	1.520	1.326	$536	$304	56.7%	4.73	4.10	31.39	24.13
GPD	0.95	7,500	$375	$718	$343	91.3%	$356	-$19	-5.1%	$1,346	$552	41.0%	$1,301	$552	42.4%	$3,716	$1,718	46.2%	1.812	1.549	$577	$296	51.4%	5.58	4.78	41.94	32.11
GPD	0.875	47,500	$1,106	$1,939	$833	75.3%	$1,037	-$69	-6.3%	$3,256	$1,460	44.8%	$3,147	$1,458	46.3%	$9,942	$4,819	48.5%	1.623	1.406	$1,626	$867	53.3%	5.05	4.33	35.94	27.15
GPD	0.95	25,000	$1,251	$2,390	$1,139	91.0%	$1,186	-$66	-5.2%	$4,483	$1,839	41.0%	$4,336	$1,838	42.4%	$12,387	$5,726	46.2%	1.815	1.550	$1,924	$988	51.3%	5.58	4.79	42.02	32.20
GPD	0.925	50,000	$1,938	$3,500	$1,563	80.7%	$1,787	-$150	-7.8%	$6,195	$2,636	42.6%	$5,994	$2,632	43.9%	$18,064	$8,640	47.8%	1.712	1.473	$2,948	$1,533	52.0%	5.37	4.59	40.26	30.46
GPD	0.99	27,500	$2,076	$4,014	$1,938	93.4%	$1,923	-$153	-7.4%	$7,557	$2,993	39.6%	$7,304	$2,989	40.9%	$21,356	$9,449	44.2%	1.820	1.554	$3,265	$1,623	49.7%	5.60	4.78	43.16	32.90
TGPD	0.775	33,500	$351	$591	$240	68.2%	$349	-$2	-0.6%	$971	$486	50.0%	$941	$486	51.6%	$3,024	$1,629	53.9%	1.592	1.391	$497	$298	60.0%	5.39	4.53	43.82	31.38
TGPD**	0.8	25,000	$361	$553	$192	53.2%	$323	-$38	-10.5%	$855	$434	50.8%	$833	$433	51.9%	$2,475	$1,336	54.0%	1.507	1.339	$461	$270	58.7%	4.86	4.45	33.19	28.30
TGPD	0.8675	50,000	$1,267	$2,207	$940	74.2%	$1,211	-$55	-4.4%	$4,158	$1,932	46.5%	$4,050	$1,931	47.7%	$12,655	$6,284	49.7%	1.835	1.594	$1,730	$955	55.2%	6.99	6.09	74.85	58.29
TGPD	0.91	31,000	$1,334	$2,322	$988	74.1%	$1,231	-$103	-7.7%	$3,918	$1,763	45.0%	$3,791	$1,760	46.4%	$11,971	$6,065	50.7%	1.633	1.430	$1,901	$1,023	53.8%	4.62	4.16	27.53	22.73
TGPD	0.92	47,500	$2,088	$4,035	$1,947	93.2%	$2,091	$3	0.1%	$8,964	$3,843	42.9%	$8,750	$3,843	43.9%	$20,662	$10,129	49.0%	2.169	1.838	$3,159	$1,688	53.4%	10.01	8.64	156.71	124.03
TGPD	0.95	35,000	$2,227	$4,133	$1,906	85.5%	$2,101	-$126	-5.7%	$8,286	$3,492	42.1%	$8,064	$3,490	43.3%	$21,228	$10,201	48.1%	1.951	1.661	$3,294	$1,706	51.8%	7.38	6.22	81.17	59.19

*NOTE: #simulations = 1,000; λ = 25 for 10 years so n ~ 250; α=0.9997

** 10% contamination instead of 5%.

25% contamination used instead of 5%.

TABLE F11a

RCE vs. LDA for Regulatory Capital Estimation Under 5% Right, 5%Left-Tail Contamination ($m, λ=25)*

Severity Dist.	Parm1	Parm2	True RCap	Mean MLE RCap	MLE Bias	MLE Bias%	Mean RCE RCap	RCE Bias	RCE Bias%	RMSE MLE RCap	RMSE RCE RCap	RMSE RCap RCE/MLE	StdDev MLE RCap	StdDev RCE RCap	StdDev RCap RCE/MLE	95%CIs MLE RCap	95%CIs RCE RCap	95%CIs RCap RCE/MLE	CV MLE	CV RCE	IQR MLE RCap	IQR RCE RCap	IQR RCap RCE/MLE	Skew MLE RCap	Skew RCE RCap	Kurtosis MLE RCap	Kurtosis RCE RCap
	μ	σ																									
LogN	10	2	$63	$67	$4	6.7%	$63	$0	0.6%	$26	$24	91.9%	$26	$24	93.1%	$99	$93	93.6%	0.390	0.385	$31	$29	92.3%	1.46	1.44	6.08	5.93
LogN	7.7	2.55	$53	$59	$6	11.7%	$54	$1	1.7%	$32	$28	87.9%	$31	$28	89.6%	$114	$103	90.0%	0.523	0.515	$35	$31	89.5%	2.22	2.18	14.12	13.53
LogN	10.4	2.5	$649	$721	$73	11.2%	$659	$10	1.6%	$375	$331	88.3%	$368	$331	90.0%	$1,365	$1,232	90.3%	0.510	0.503	$416	$374	89.8%	2.14	2.10	13.12	12.60
LogN	9.27	2.77	$603	$688	$85	14.1%	$616	$14	2.3%	$407	$351	86.2%	$398	$350	88.1%	$1,452	$1,285	88.5%	0.579	0.569	$431	$381	88.4%	2.62	2.55	19.37	18.42
LogN	10.75	2.7	$2,012	$2,280	$269	13.3%	$2,054	$42	2.1%	$1,306	$1,133	86.8%	$1,278	$1,133	88.6%	$4,685	$4,168	88.9%	0.561	0.551	$1,400	$1,242	88.8%	2.49	2.43	17.53	16.72
LogN	9.63	2.97	$1,893	$2,207	$313	16.5%	$1,947	$54	2.8%	$1,428	$1,208	84.6%	$1,394	$1,207	86.6%	$5,017	$4,369	87.1%	0.632	0.620	$1,456	$1,269	87.2%	3.04	2.95	25.61	24.18
TLogN	10.2	1.95	$76	$89	$13	17.2%	$78	$2	2.7%	$60	$47	77.8%	$59	$47	79.6%	$226	$178	78.7%	0.656	0.596	$55	$47	84.7%	2.83	2.51	13.49	10.61
TLogN	9	2.2	$76	$102	$26	34.7%	$78	$2	2.9%	$104	$57	55.1%	$101	$57	56.9%	$320	$206	64.3%	0.986	0.734	$72	$54	74.9%	4.64	2.71	37.03	12.00
TLogN	10.7	2.385	$670	$863	$193	28.9%	$709	$39	5.8%	$802	$557	69.5%	$778	$555	71.4%	$2,463	$1,880	76.3%	0.901	0.784	$587	$472	80.4%	4.86	3.68	43.70	24.26
TLogN	9.4	2.65	$643	$959	$316	49.2%	$652	$9	1.4%	$1,454	$605	41.6%	$1,419	$605	42.6%	$3,583	$2,140	59.7%	1.480	0.928	$668	$481	72.1%	8.37	3.75	102.69	22.80
TLogN	11	2.6	$2,085	$2,818	$733	35.1%	$2,231	$146	7.0%	$3,125	$2,041	65.3%	$3,037	$2,036	67.0%	$8,555	$6,329	74.0%	1.078	0.912	$1,948	$1,542	79.1%	6.34	4.91	66.54	42.98
TLogN	10	2.8	$1,956	$2,844	$888	45.4%	$2,008	$53	2.7%	$3,429	$1,866	54.4%	$3,312	$1,866	56.3%	$11,047	$6,805	61.6%	1.165	0.929	$2,233	$1,578	70.7%	4.38	2.98	30.22	12.87
	a	b																									
Logg	24	2.65	$85	$100	$15	17.6%	$90	$5	5.6%	$70	$60	85.6%	$69	$60	87.4%	$248	$218	87.9%	0.683	0.664	$65	$57	88.1%	2.94	2.84	14.19	13.33
Logg	33	3.3	$100	$112	$12	12.3%	$103	$3	3.1%	$59	$52	88.2%	$58	$52	90.0%	$219	$196	89.4%	0.521	0.510	$62	$57	91.8%	1.66	1.61	4.00	3.79
Logg	25	2.5	$444	$524	$80	18.1%	$464	$20	4.6%	$356	$299	84.1%	$347	$299	86.2%	$1,326	$1,139	85.9%	0.661	0.644	$334	$297	89.1%	2.05	1.97	6.21	5.61
Logg	34.5	3.15	$448	$506	$58	13.0%	$460	$12	2.7%	$299	$262	87.6%	$293	$261	89.2%	$1,118	$984	88.0%	0.579	0.568	$310	$270	87.2%	2.28	2.26	10.08	10.08
Logg	25.25	2.45	$766	$916	$149	19.5%	$806	$40	5.2%	$704	$589	83.6%	$688	$587	85.4%	$2,332	$2,017	86.5%	0.751	0.729	$613	$540	88.1%	3.57	3.43	21.15	19.67
Logg	34.7	3.07	$818	$928	$110	13.4%	$840	$22	2.6%	$559	$485	86.9%	$548	$485	88.5%	$2,068	$1,816	87.8%	0.590	0.578	$549	$494	89.9%	2.18	2.13	7.60	7.27
TLogg	23.5	2.65	$124	$230	$106	85.5%	$145	$21	17.0%	$543	$445	81.9%	$533	$444	83.4%	$938	$530	56.5%	2.319	3.067	$156	$86	55.5%	16.99	23.30	400.47	645.03
TLogg	33	3.3	$130	$178	$49	37.6%	$130	$1	0.7%	$244	$118	48.5%	$239	$118	49.5%	$573	$320	55.8%	1.338	0.905	$121	$90	74.4%	9.61	6.94	142.23	87.47
TLogg	24.5	2.5	$495	$872	$377	76.2%	$457	-$38	-7.6%	$1,581	$537	34.0%	$1,536	$536	34.9%	$3,573	$1,485	41.6%	1.760	1.171	$692	$328	47.4%	7.61	6.54	80.18	66.51
TLogg	34.5	3.15	$510	$658	$148	29.0%	$552	$42	8.3%	$613	$454	74.2%	$595	$452	76.1%	$2,053	$1,589	77.4%	0.904	0.820	$518	$423	81.8%	3.53	3.20	21.50	18.18
TLogg	24.75	2.45	$801	$1,336	$535	66.8%	$752	-$49	-6.2%	$2,541	$938	36.9%	$2,484	$936	37.7%	$5,528	$2,139	38.7%	1.859	1.246	$1,012	$542	53.6%	11.34	11.46	187.34	210.41
TLogg	34.6	3.07	$867	$1,097	$231	26.6%	$936	$70	8.0%	$906	$696	76.9%	$876	$693	79.1%	$3,270	$2,611	79.9%	0.798	0.740	$813	$676	83.1%	2.41	2.23	8.50	7.56
	ξ	θ																									
GPD	0.8	35,000	$149	$235	$87	58.3%	$153	$4	3.0%	$303	$169	55.8%	$291	$169	58.2%	$999	$606	60.7%	1.235	1.105	$199	$127	64.0%	3.78	3.36	20.19	16.18
GPD	0.95	7,500	$121	$215	$94	78.1%	$125	$4	3.7%	$331	$162	49.0%	$317	$162	51.1%	$1,005	$545	54.2%	1.473	1.293	$181	$104	57.6%	4.69	4.05	31.66	23.99
GPD	0.875	47,500	$391	$643	$252	64.5%	$397	$6	1.6%	$880	$464	52.7%	$843	$464	55.0%	$2,918	$1,594	54.6%	1.310	1.167	$542	$334	61.6%	3.96	3.48	22.28	17.37
GPD	0.95	25,000	$403	$712	$309	76.8%	$414	$12	2.9%	$1,096	$536	48.9%	$1,051	$536	51.0%	$3,350	$1,806	53.9%	1.476	1.294	$603	$347	57.6%	4.74	4.10	32.37	24.63
GPD	0.925	50,000	$643	$1,082	$439	68.2%	$645	$2	0.2%	$1,554	$786	50.6%	$1,491	$786	52.7%	$4,946	$2,709	54.8%	1.377	1.219	$933	$552	59.1%	4.26	3.72	25.82	19.97
GPD	0.99	27,500	$636	$1,133	$497	78.3%	$642	$6	1.0%	$1,731	$825	47.7%	$1,657	$825	49.8%	$5,462	$2,788	51.0%	1.463	1.286	$992	$556	56.0%	4.49	3.91	28.47	22.24
TGPD	0.775	33,500	$141	$219	$78	55.3%	$147	$5	3.9%	$286	$165	57.6%	$276	$165	59.8%	$880	$585	66.4%	1.258	1.125	$179	$118	65.9%	4.09	3.70	24.33	20.59
TGPD	0.8	25,000	$140	$221	$80	57.2%	$145	$4	3.2%	$307	$172	56.2%	$296	$172	58.2%	$934	$557	59.7%	1.342	1.189	$178	$115	64.8%	4.50	4.01	28.12	22.80
TGPD	0.8675	50,000	$452	$770	$318	70.3%	$482	$30	6.6%	$1,341	$698	52.0%	$1,302	$697	53.5%	$3,510	$2,061	58.7%	1.692	1.447	$628	$392	62.5%	10.81	8.52	199.20	134.44
TGPD	0.91	31,000	$451	$759	$307	68.1%	$464	$12	2.7%	$1,126	$581	51.6%	$1,083	$581	53.7%	$3,188	$1,862	58.4%	1.428	1.254	$672	$410	61.1%	5.23	4.38	42.92	30.28
TGPD	0.92	47,500	$698	$1,138	$441	63.1%	$698	$0	0.0%	$1,565	$835	53.3%	$1,501	$835	55.6%	$4,788	$2,799	58.5%	1.319	1.196	$963	$583	60.5%	3.88	3.55	22.39	18.89
TGPD	0.95	35,000	$717	$1,268	$551	76.9%	$753	$36	5.0%	$1,938	$986	50.9%	$1,858	$986	53.0%	$6,149	$3,326	54.1%	1.465	1.309	$1,035	$614	59.4%	4.57	4.04	31.88	24.64

*NOTE: #simulations = 1,000; λ = 25 for 10 years so n ~ 250; α=0.999

TABLE F11b

RCE vs. LDA for Economic Capital Estimation Under 5% Right, 5%Left-Tail Contamination ($m, λ=25)*

Severity Dist.	Parm1	Parm2	True ECap	Mean MLE ECap	MLE Bias	MLE Bias%	Mean RCE ECap	RCE Bias	RCE Bias%	RMSE MLE ECap	RMSE RCE ECap	RMSE ECap RCE/MLE	StdDev MLE ECap	StdDev RCE ECap	StdDev ECap RCE/MLE	95%CIs MLE ECap	95%CIs RCE ECap	95%CIs ECap RCE/MLE	CV MLE	CV RCE	IQR MLE ECap	IQR RCE ECap	IQR ECap RCE/MLE	Skew MLE ECap	Skew RCE ECap	Kurtosis MLE ECap	Kurtosis RCE ECap
	μ	σ																									
LogN	10	2	$107	$115	**$8**	**7.8%**	$108	**$1**	**1.2%**	$49	$45	**91.5%**	$49	$45	92.8%	$181	$168	**92.6%**	0.422	0.417	$57	$54	93.5%	1.65	1.62	7.91	7.72
LogN	7.7	2.55	$107	$121	**$14**	**13.4%**	$109	**$3**	**2.7%**	$70	$61	**87.3%**	$68	$61	89.1%	$247	$220	**89.2%**	0.565	0.556	$74	$67	90.5%	2.58	2.52	19.00	18.19
LogN	10.4	2.5	$1,286	$1,451	**$165**	**12.8%**	$1,319	**$33**	**2.6%**	$817	$717	**87.7%**	$800	$716	89.5%	$2,902	$2,598	**89.5%**	0.551	0.543	$876	$796	90.8%	2.47	2.42	17.60	16.88
LogN	9.27	2.77	$1,293	$1,502	**$209**	**16.2%**	$1,337	**$44**	**3.4%**	$964	$824	**85.5%**	$941	$823	87.4%	$3,343	$2,930	**87.7%**	0.627	0.615	$981	$874	89.1%	3.07	2.99	26.42	25.10
LogN	10.75	2.7	$4,230	$4,876	**$646**	**15.3%**	$4,365	**$135**	**3.2%**	$3,027	$2,605	**86.1%**	$2,958	$2,602	88.0%	$10,567	$9,315	**88.2%**	0.607	0.596	$3,126	$2,800	89.6%	2.90	2.83	23.82	22.68
LogN	9.63	2.97	$4,303	$5,118	**$815**	**18.9%**	$4,482	**$179**	**4.2%**	$3,603	$3,017	**83.7%**	$3,510	$3,011	85.8%	$12,246	$10,555	**86.2%**	0.686	0.672	$3,515	$3,081	87.6%	3.61	3.49	35.34	33.32
TLogN	10.2	1.95	$126	$153	**$27**	**21.7%**	$130	**$5**	**3.9%**	$119	$88	**74.5%**	$116	$88	76.4%	$445	$335	**75.2%**	0.756	0.677	$99	$83	83.7%	3.23	2.81	17.49	13.18
TLogN	9	2.2	$133	$194	**$61**	**45.8%**	$137	**$4**	**3.3%**	$240	$111	**46.3%**	$232	$111	47.8%	$694	$398	**57.3%**	1.199	0.808	$140	$101	72.4%	5.76	2.70	56.19	11.14
TLogN	10.7	2.385	$1,267	$1,722	**$455**	**35.9%**	$1,362	**$95**	**7.5%**	$1,874	$1,212	**64.7%**	$1,817	$1,208	66.5%	$5,385	$3,983	**74.0%**	1.055	0.887	$1,243	$961	77.3%	5.98	4.21	64.76	31.17
TLogN	9.4	2.65	$1,297	$2,154	**$857**	**66.0%**	$1,321	**$24**	**1.9%**	$4,206	$1,364	**32.4%**	$4,117	$1,364	33.1%	$8,757	$4,740	**54.1%**	1.912	1.032	$1,498	$1,010	67.4%	10.69	4.06	156.41	26.36
TLogN	11	2.6	$4,208	$6,059	**$1,850**	**44.0%**	$4,589	**$381**	**9.1%**	$7,956	$4,782	**60.1%**	$7,738	$4,766	61.6%	$19,963	$14,205	**71.2%**	1.277	1.039	$4,282	$3,313	77.4%	7.53	5.67	88.00	54.80
TLogN	10	2.8	$4,145	$6,579	**$2,434**	**58.7%**	$4,289	**$144**	**3.5%**	$9,318	$4,406	**47.3%**	$8,995	$4,404	49.0%	$28,334	$15,565	**54.9%**	1.367	1.027	$5,206	$3,509	67.4%	5.23	3.22	42.69	14.97
	a	b																									
Logg	24	2.65	$192	$234	**$42**	**21.6%**	$207	**$14**	**7.4%**	$185	$155	**83.7%**	$180	$154	85.6%	$650	$550	**84.7%**	0.770	0.746	$158	$139	87.9%	3.28	3.16	17.25	16.10
Logg	33	3.3	$203	$234	**$31**	**15.0%**	$212	**$9**	**4.3%**	$139	$120	**86.7%**	$135	$120	88.6%	$509	$450	**88.4%**	0.579	0.566	$137	$124	90.4%	1.84	1.79	4.97	4.67
Logg	25	2.5	$1,064	$1,301	**$238**	**22.3%**	$1,130	**$66**	**6.2%**	$989	$811	**82.0%**	$960	$808	84.2%	$3,588	$3,070	**85.6%**	0.738	0.715	$874	$760	86.9%	2.29	2.19	7.83	6.99
Logg	34.5	3.15	$960	$1,114	**$153**	**16.0%**	$999	**$39**	**4.0%**	$735	$632	**86.0%**	$719	$631	87.8%	$2,722	$2,369	**87.0%**	0.646	0.632	$717	$632	88.1%	2.56	2.53	12.51	12.41
Logg	25.25	2.45	$1,877	$2,329	**$452**	**24.1%**	$2,009	**$132**	**7.0%**	$2,037	$1,657	**81.4%**	$1,986	$1,652	83.2%	$6,463	$5,481	**84.8%**	0.853	0.822	$1,653	$1,410	85.3%	4.05	3.87	26.26	24.23
Logg	34.7	3.07	$1,794	$2,090	**$296**	**16.5%**	$1,864	**$70**	**3.9%**	$1,407	$1,198	**85.2%**	$1,375	$1,196	87.0%	$5,191	$4,514	**87.0%**	0.658	0.642	$1,343	$1,187	88.4%	2.42	2.35	9.27	8.79
TLogg	23.5	2.65	$271	$639	**$368**	**136.1%**	$340	**$69**	**25.6%**	$2,465	$2,102	**85.3%**	$2,437	$2,101	86.2%	$2,863	$1,280	**44.7%**	3.813	6.180	$396	$178	44.8%	23.14	28.29	638.93	855.18
TLogg	33	3.3	$261	$400	**$139**	**53.3%**	$247	**-$14**	**-5.5%**	$742	$256	**34.4%**	$729	$255	35.0%	$1,466	$611	**41.7%**	1.821	1.034	$277	$178	64.4%	12.49	8.97	218.19	133.34
TLogg	24.5	2.5	$1,164	$2,455	**$1,291**	**110.9%**	$898	**-$267**	**-22.9%**	$5,805	$1,286	**22.1%**	$5,659	$1,258	22.2%	$10,711	$3,249	**30.3%**	2.305	1.401	$1,887	$618	32.8%	9.04	8.17	108.19	96.40
TLogg	34.5	3.15	$1,086	$1,498	**$412**	**37.9%**	$1,190	**$104**	**9.6%**	$1,627	$1,097	**67.4%**	$1,574	$1,092	69.4%	$5,400	$3,767	**69.8%**	1.051	0.918	$1,241	$964	77.7%	4.11	3.57	28.75	22.34
TLogg	24.75	2.45	$1,928	$3,815	**$1,887**	**97.9%**	$1,573	**-$356**	**-18.4%**	$10,543	$2,702	**25.6%**	$10,373	$2,678	25.8%	$17,336	$4,406	**25.4%**	2.719	1.703	$2,733	$1,143	41.8%	15.79	18.01	328.59	435.49
TLogg	34.6	3.07	$1,892	$2,537	**$645**	**34.1%**	$2,074	**$182**	**9.6%**	$2,399	$1,719	**71.7%**	$2,311	$1,710	74.0%	$8,524	$6,382	**74.9%**	0.911	0.824	$1,968	$1,564	79.5%	2.71	2.44	10.59	9.12
	ξ	θ																									
GPD	0.8	35,000	$382	$704	**$321**	**83.9%**	$399	**$17**	**4.4%**	$1,109	$528	**47.6%**	$1,061	$528	49.7%	$3,502	$1,831	**52.3%**	1.508	1.323	$597	$340	57.0%	4.51	3.93	28.12	21.78
GPD	0.95	7,500	$375	$801	**$426**	**113.4%**	$394	**$19**	**5.1%**	$1,514	$613	**40.5%**	$1,452	$613	42.2%	$4,326	$1,989	**46.0%**	1.813	1.554	$660	$330	49.9%	5.58	4.77	43.03	32.29
GPD	0.875	47,500	$1,106	$2,137	**$1,031**	**93.2%**	$1,135	**$29**	**2.6%**	$3,579	$1,586	**44.3%**	$3,427	$1,585	46.3%	$11,343	$5,256	**46.3%**	1.604	1.397	$1,842	$974	52.9%	4.79	4.14	31.97	24.38
GPD	0.95	25,000	$1,251	$2,650	**$1,398**	**111.8%**	$1,304	**$53**	**4.2%**	$5,018	$2,031	**40.5%**	$4,819	$2,031	42.1%	$14,421	$6,412	**44.5%**	1.819	1.557	$2,166	$1,099	50.7%	5.64	4.83	43.87	33.08
GPD	0.925	50,000	$1,938	$3,853	**$1,916**	**98.9%**	$1,955	**$18**	**0.9%**	$6,795	$2,860	**42.1%**	$6,519	$2,860	43.9%	$19,904	$9,416	**47.3%**	1.692	1.463	$3,273	$1,688	51.6%	5.15	4.44	36.70	27.99
GPD	0.99	27,500	$2,076	$4,437	**$2,361**	**113.7%**	$2,111	**$35**	**1.7%**	$8,311	$3,257	**39.2%**	$7,968	$3,257	40.9%	$23,923	$10,500	**43.9%**	1.796	1.543	$3,767	$1,816	48.2%	5.39	4.64	39.80	30.52
TGPD	0.775	33,500	$351	$634	**$283**	**80.5%**	$372	**$21**	**5.8%**	$1,022	$506	**49.5%**	$982	$505	51.5%	$2,935	$1,677	**57.1%**	1.549	1.359	$529	$309	58.5%	4.86	4.35	33.45	27.94
TGPD	0.8	25,000	$361	$663	**$302**	**83.7%**	$379	**$18**	**5.0%**	$1,147	$548	**47.7%**	$1,106	$547	49.5%	$3,266	$1,656	**50.7%**	1.669	1.444	$532	$307	57.8%	5.34	4.68	38.41	30.31
TGPD	0.8675	50,000	$1,267	$2,582	**$1,316**	**103.9%**	$1,390	**$123**	**9.7%**	$6,016	$2,577	**42.8%**	$5,871	$2,574	43.8%	$13,331	$6,710	**50.3%**	2.274	1.852	$2,084	$1,130	54.3%	14.74	11.50	323.63	219.01
TGPD	0.91	31,000	$1,334	$2,651	**$1,317**	**98.8%**	$1,384	**$50**	**3.8%**	$4,916	$2,106	**42.8%**	$4,736	$2,105	44.4%	$12,491	$6,249	**50.0%**	1.787	1.521	$2,333	$1,224	52.5%	6.60	5.39	65.93	45.09
TGPD	0.92	47,500	$2,088	$3,987	**$1,899**	**90.9%**	$2,099	**$11**	**0.5%**	$6,618	$2,984	**45.1%**	$6,340	$2,984	47.1%	$18,525	$9,553	**51.6%**	1.590	1.422	$3,366	$1,782	52.9%	4.61	4.21	31.11	26.22
TGPD	0.95	35,000	$2,227	$4,713	**$2,486**	**111.6%**	$2,377	**$150**	**6.7%**	$8,807	$3,742	**42.5%**	$8,449	$3,739	44.3%	$26,750	$11,880	**44.4%**	1.793	1.573	$3,800	$1,960	51.6%	5.55	4.83	46.92	35.21

*NOTE: #simulations = 1,000; λ = 25 for 10 years so n ~ 250; α=0.9997